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On particle collisions during gravitational collapse of Vaidya spacetimes

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Abstract. The center-of-mass energy can be arbitrarily high in Schwarzschild spacetime if one considers the front collision of two particles, one of which moves along the so-called white hole geodesics and the other one along the black hole geodesic. This process can take place if one considers the gravitational collapse model. In this paper, we consider the well-known naked singularity formation in the Vaidya spacetime and investigate the question about two particle collision near the boundary of the collapsing cloud. The center-of-mass energy of the front collision is considered. One particle moves away from the naked singularity and the other one falls onto a collapsing cloud. We show that the center-of-mass energy grows unboundly if the collision takes place in the vicinity of the conformal Killing horizon.

Keywords: gravitational collapse, particles collision, Vaidya spacetime, naked singularity, conformal symmetry

Introduction

The center-of-mass energy of two particles collision can grow unboundly in the Kerr spacetime (Banados et al. 2009) if one of the particles is fine-tunned (the so-called critical particle (Tanatarov, Zaslavskii 2013). This effect was first proposed by Bañados, Silk and West and is called the BSW effect. The original version of this effect declares absence of the unbound energies in Schwarzschild and Reissner-Nordström spacetimes. However, it was shown that this effect is possible in Reissner-Nordström-anti-de Sitter spacetime (Zaslavskii 2012a). Despite the unbound center-of-mass energy, a distant observer will measure small amount of the energy due to this process in the Kerr spacetime (Harada et al. 2012) and an escaping particle will be able to carry away arbitrarely large amount of energy in Reissner-Nordström case (Zaslavskii 2012b).

In spite of the fact that the BSW effect is absent in the Schwarzschild spacetime, one can still obtain the unbound center-of-mass energy of two colliding particles (Grib, Pavlov 2015). Due to geodesic completeness, there must be geodesics which appear in our Universe from the region inside the gravitational radius, i.e., the so-called white hole geodesics. For example, geodesics for particles with negative energy in the Kerr metric are such geodesics (Grib et al. 2014; Vertogradov 2015). One can imagine the following situation: the first particle moves along the white hole geodesic away from the gravitational radius and the second particle moves along the black hole geodesics falling onto the black hole. As a result, one can observe the front collision in the vicinity of the event horizon and due to this process the center-of-mass energy can grow unboundly. The problem is that the Schwarzschild black hole is an eternal one and if one follows the geodesic back, then it must appear from the collapsing cloud. So, to understand the front collision in the vicinity of the event horizon in the Schwarzschild spacetime, one must consider the gravitational collapse problem. The nature of the white hole geodesics can be explained by the naked singularity formation due to the gravitational collapse problem (Vertogradov 2021). The outcome of the gravitational collapse is not limited to a black hole but also a naked singularity (Dey et al. 2022; Joshi 2007; Joshi, Malafarina 2011). The naked singularity formation in the Vaidya spacetime has been considered in (Dwivedi, Joshi 1989). The gravitational collapse of the generalized Vaidya spacetime and the naked singularity formation has been investigated in (Mkenyeleye et al. 2014; Vertogradov 2016a; 2016b; 2018). In the case of the eternal naked singularity formation in the Vaidya spacetime (Vertogradov 2018; 2022a) the unbound center-of-mass energy is possible only in the vicinity of the singularity.

In this paper, we consider the following model: a particle moves along the non-spacelike, future-directed geodesic which terminates at the naked singularity in the past. When the apparent horizon forms, the particle is in the vicinity of the apparent horizon and outside it. At this time, the second particle, moving along the black hole geodesic, falls onto the black hole. As a result, we have a front collision of two particles. We estimate the center-of-mass energy of this process and find out where this process should take place to results in unbound energy collision.

This paper is organized as follows: in Sec. 2 we consider the well-known gravitational collapse model of the Vaidya spacetime and show the naked singularity formation. We also show that the geodesics can originate at this singularity. In Sec. 3 we introduce the coordinate transformation and consider the Vaidya spacetime in conformally static coordinates. In this case we investigate the center-of-mass energy of the two particle front collision. Sec. 4 is the conclusion.

The system of units $G = c = 1$ will be used throughout the paper. We use the signature $-, +, +, +$.

Naked singularity formation in the Vaidya spacetime

The Vaidya metric (Vaidya 1951) describes a dynamical spacetime instead of a static spacetime as the Schwarzschild or Reissner-Nordström metrics do. In the real world, astronomical bodies gain mass when they absorb radiation and they lose mass when they emit radiation. This means that the space-time around them is time-dependent. Papapetroo (Papapetrou 1985) showed that the Vaidya spacetime violates the cosmic censorship conjecture and contains a naked singularity. The line element in Eddington-Finkelstein coordinates has the following form:

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right)dv^2 + 2dvdr + r^2 d\omega^2. \tag{1}$$

Here $M(v)$ is the time-depended mass of the black hole, $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$ is the metric on unit sphere.

The apparent horizon equation is given by (Poisson 2004):

$$r_{ah} = 2M(v)\ . \tag{2}$$

The first shell collapses at $r = 0$ at the time $v = 0$ and the singularity forms at this time. The singularity is naked if at the time of the singularity formation $v = 0$ the apparent horizon doesn't form and there is a family of non-spacelike, future-directed geodesics which terminate at the central singularity in the past. Let's prove the last statement. For this purpose, we define the mass function as:

$$M(v) = \mu v,\ \ \mu > 0\ . \tag{3}$$

Here μ is a positive constant. Here, we just show that a naked singularity is possible. See (Dwivedi, Joshi 1989) for a substantive investigation of this model.

To prove the existence of a family of non-spacelike, future-directed geodesics which terminate at the central singularity in the past, one should consider the null radial geodesic, which, for the metric (1) with the mass condition (3) has the following form:

$$\frac{dv}{dr} = \frac{2r}{r - 2\mu v}\ . \tag{4}$$

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The solution $v = const$ doesn't suit us because for infalling matter $v = const$ corresponds to ingoing geodesics and we are interested in outgoing ones. So, (4) corresponds to outgoing geodesic if the following condition is held:

$$\lim_{v\to 0\,, r\to 0} \frac{dv}{dr} = X_0 > 0\ . \tag{5}$$

If the value X_0 is positive and finite, then the geodesic (4) is outgoing one. Let's consider the limit in (4):

$$X_0 = \frac{2}{1-2\mu X_0}\ . \tag{6}$$

From this equation we obtain:

$$X_0^{\pm} = \frac{1\pm\sqrt{1-16\mu}}{4\mu}\ . \tag{7}$$

From this equation, one can see that in the case of the linear mass function and if $\mu < \frac{1}{16}$, then the outcome of the gravitational collapse might be the naked singularity formation. Hence, there might be particles which move away the singularity. Now we are ready to consider the front collision of two particles.

Front collision effect in the Vaidya spacetime

The Vaidya spacetime (1) is time-depended and because of it one has only one conserved quantity—the angular momentum L. In the general case, the Vaidya spacetime doesn't possess any additional symmetry. However, for the particular choice of the mass function, the metric (1) admits the conformal Killing vector (Ojako et al. 2020). In this case, M must have the following form (Nielsen 2014):

$$M\left(v\right) = \mu v,\ \mu > 0\ . \tag{8}$$

Where μ is a positive constant. As we found out in the previous section, if $\mu < \frac{1}{16}$, the gravitational collapse might end with the naked singularity formation. Further in the paper we impose the condition $\mu < \frac{1}{16}$ because we are interested in the temporal naked singularity formation. If we take into account this condition and (8), then by a coordinate transformation (Solanki, Perlick 2022):

$$v = r_0 e^{\frac{t}{r_0}}\ ,\quad r = Re^{\frac{t}{r_0}}\ . \tag{9}$$

one obtains the Vaidya metric in conformally static coordinates:

$$ds^2 = e^{\frac{2t}{r_0}}\left[-\left(1-\frac{2\mu r_0}{R}-\frac{2R}{r_0}\right)dt^2 + 2dtdR + R^2 d\Omega^2\right]. \tag{10}$$

We will consider the movement in the equatorial plane $\theta = \frac{\pi}{2}$. The metric (10) admits the conformal Killing vector $\frac{d}{dt}$, which is timelike in the region:

$$1-\frac{2\mu r_0}{R}-\frac{2R}{r_0} > 0\ . \tag{11}$$

It means that one has the conserved energy along null geodesics in the region (11), i. e.:

$$E = e^{\frac{2t}{r_0}}\left(1-\frac{2\mu r_0}{R}-\frac{2R}{r_0}\right)\frac{dt}{d\lambda} - e^{\frac{2t}{r_0}}\frac{dR}{d\lambda}\ . \tag{12}$$

The angular momentum L has the following form:

$$L = e^{\frac{2t}{r_0}} R^2 \frac{d\varphi}{d\lambda}\ . \tag{13}$$

The problem is that the energy (12) is not a constant of motion along timelike geodesics. Fortunately, the conformal Killing vector $\frac{d}{dt}$ is the homothetic Killing vector (Blau 2022) and this fact allows us to find another constant of motion along a timelike geodesic:

$$\varepsilon = E - \lambda \cdot \tag{14}$$

Despite the fact that this quantity (14) depends on the affine parameter λ, it is conserved charge along any geodesics.

To find the $\frac{dR}{d\lambda}$ component of the four velocity u^i, one should substitute (14) and (13) into the timelike condition $g_{ik}\, u^i\, u^k = -1$. Thus, one obtains:

$$e^{\frac{4t}{r_0}}\left(\frac{dR}{d\lambda}\right)^2 = E^2 - e^{\frac{2t}{r_0}}\left(1-\frac{2\mu r_0}{R}-\frac{2R}{r_0}\right)\left(\frac{L^2}{r^2 e^{\frac{2t}{r_0}}}+1\right) = e^{\frac{4t}{r_0}} P_R^2 \ . \tag{15}$$

Where $P_R = P_R\,(R,t)$ is some positive function. According to the BSW effect (Banados et al. 2009), the energy of the centre of mass $E_{c.m.}$ of the two colliding particles for the extremal Kerr black hole can grow unboundly. For this purpose, one of the particles must be a critical one (Tanatarov, Zaslavskii 2013). In the Schwarzschild spacetime, the energy $E_{c.m.}$ is finite according to the original proposal. However, if we consider, in the Schwarzschild spacetime, the collision of two particles one of which moves along the white hole geodesic from the gravitational radius and the other one moves along the black hole geodesic and falls onto the black hole (Grib, Pavlov 2015), then the energy $E_{c.m.}$ of the collision can be unbound. The problem is that the Schwarzschild metric describes the eternal black hole and the white hole geodesic appears from the region outside the white hole gravitational radius in past infinity. However, if one considers the physically relevant model, then prolonging the white hole geodesic into the past, one can see that it appears from the collapsing cloud of the matter. So, to understand this analogue of the BSW effect one, first of all, should consider the gravitational collapse model which, in the case of the Vaidya spacetime, was done in the previous section. We have proved that if $\mu < \frac{1}{16}$, the gravitational collapse might end with the naked singularity formation. For our model it means that there is a family of non-spacelike future-directed geodesics which terminate at the central singularity in the past. Let's consider the following situation: particle moves along such geodesic and when the apparent horizon forms, this particle is in the vicinity of this horizon and outside it. At this time, particle 2, which falls onto the black hole, collides with particle 1. Let's calculate if the unbounded energy $E_{c.m.}$ of the collision is possible and where this collision should take place.

For simplisity, let's consider the collision of two particles with same mass m_0. In this case, the energy $E_{c.m.}$ is given by:

$$E_{c.m.} = m_0\sqrt{2}\sqrt{1-g_{ik}u_1^i u_2^k} \ . \tag{16}$$

Where and are the four velocities of particles and respectivelly. Substituting (12), (13) and (15) into (16), one obtains:

$$\frac{E_{c.m.}^2}{2m_0} = 1 + \frac{E_1\left(E_2 + e^{\frac{2t}{r_0}} P_{2R}\right)}{e^{\frac{2t}{r_0}}\left(1-\frac{2\mu r_0}{R}-\frac{2R}{r_0}\right)} - \frac{e^{\frac{2t}{r_0}}\left(E_1 + e^{\frac{2t}{r_0}} P_{1R}\right) P_{2R}}{1-\frac{2\mu r_0}{R}-\frac{2R}{r_0}} - \frac{L_1 L_2}{e^{\frac{2t}{r_0}} R^2} \ . \tag{17}$$

Note that only the second and third terms in the right-hand side can give us the unbound energy $E_{c.m.}$. It is possible if:

$$1-\frac{2\mu r_0}{R_{kh}}-\frac{2R_{kh}}{r_0} = 0 \ . \tag{18}$$

Where $R = R_{kh}$ is the location of the conformal Killing horizon. Also, one should note that for outgoing particle $1 - P_{1R} > 0$, for ingoing particle $2 - P_{2R} < 0$. So, we conclude that both considered terms are positive if we consider the region (11) of the timelike conformal Killing vector $\frac{d}{dt}$ and we should prove that one of them grows unboundly when $R \to R_{kh}$. To proceed, we note that:

$$\lim_{R \to R_{kh}} e^{\frac{2t}{r_0}} P_{1R} = +E \ , \quad \lim_{R \to R_{kh}} e^{\frac{2t}{r_0}} P_{2R} = -E \ . \tag{19}$$

One should note that an off-diagonal term in the metric (10) might indicate that there are particles with negative energy. However, it was shown (Vertogradov 2020) that there are no particles with negative energy outside the apparent horizon and $E \geq 0$. Using this fact and by taking limit $R \to R_{kh}$, one can see that the second term in the right-hand side of (17) gives us uncertainty 0/0 and we won't consider it because if it is finite, then we can neglect it. If it is infinite, then we obtain the unbound $E_{c.m}$. However, we focus our attention on the third term in the right-hand side of (17):

$$\lim_{R \to R_{kh}} - \frac{e^{\frac{2t}{r_0}} \left(E_1 + e^{\frac{2t}{r_0}} P_{1R} \right) P_{2R}}{1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0}} = \frac{e^{2t} r_0 2 E_1 E_2}{1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0}} \to +\infty \ . \tag{20}$$

And we can see that this term (20) gives us the unbound energy $E_{c.m}$ if the collision takes place in the vicinity of the conformal Killing horizon.

Conclusions

In this paper, we have considered the front collision of two particles in the Vaidya spacetime. The metric (1) is time-depended and to consider the center-of-mass energy, one needs to introduce new coordinates which allow us to write the Vaidya spacetime in a conformally static form. It allows us to consider the following model: in the case of the linear muss function, the gravitational collapse might end up with the naked singularity. We consider the non-spacelike geodesic which originates at this naked singularity. Further, we assume that there are particles which move along this geodesic away from the central singularity. Then, the other particle falls onto a collapsing cloud and the front collision of two particles is considered at the time of the apparent horizon formation. It means that the apparent horizon forms and the particle, moving along a naked singularity geodesic, finds itself outside the trapped region, in the vicinity of the apparent horizon. We showed that the unbound center-of-mass energy is possible if the collision takes place in the vicinity of the conformal Killing horizon. Note that if we pick up the mass function as $M(v) = \mu v^n$, $n > 1$, then, of course, one has the naked singularity formation (Vertogradov 2022b). However, the singularity is gravitationally weak according to the Tipler definition (Nolan 1999; Tipler 1977) and the spacetime doesn't admit the conformal Killing vector anymore.

The unbound center-of-mass energy of the two colliding particles near the conformal Killing horizon is an expected result. One should use this horizon to define most physicaly relevant quantities. In static spacetimes, for example, one uses the Killing horizon to define the surface gravity which is associated with the Hawking temperature. The Killing horizon coinsides with the event horizon in the case of the Schwarzschild and non-extremal Reissner-Nordström black holes. However, in the dynamical case it is not an easy task to define the surface gravity (Nielsen, Yoon 2008). One can define the surface gravity on the apparent horizon but, according to Nielsen (Nielsen 2010), the apparent horizon in the Vaidya spacetime is hidden inside the event horizon, although, to define the location of the last one is also a hard task in dynamical spacetimes.

The results obtained in this paper can be easily extended to the generalized Vaidya spacetime. The naked singularity formation and the mass function conditions for this metric have been proven in (Mkenyeleye et al. 2014; Vertogradov 2016a; 2016b). The unbound center-of-mass is again expected if the front collision takes place in the vicinity of the conformal Killing horizon. However, the conformal Killing vector exists only for the following choice of the mass function:

$$M\left(v, r\right) = \mu v + \nu v^{2\alpha} r^{1-2\alpha}, \quad \mu > 0 \nu \neq 0 \, , \quad \alpha \neq \frac{1}{2} \ . \tag{20}$$

Where α ∈ [−1 ,1] (Wang, Wu 1999). Note, that for this choice of the mass function, the conformal Killing vector is the homothetic one. The generalized Vaidya spacetime admits a regular black hole solution (Hayward 2006), however, the question about the front collision in this case is still open.

Conflict of Interest

The authors declare that there is no conflict of interest, either existing or potential.

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