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On the temperature of hairy black holes

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Abstract. The gravitational decoupling method represents an extremely useful tool to obtain new solutions of the Einstein equations through minimal geometrical deformations. In this paper, we consider a hairy charged black hole obtained by the gravitational decoupling and calculate its Hawking temperature in order to compare it with the case when the hairs are ignored. We have found out that the hair, under some conditions of black hole parameters, affect the Hawking temperature and can increase it. We have also found out that the black hole temperature, in the hairy case, does not depend on the electric charge.

Keywords*:* hairy black hole, Hawking temperature, charged black hole, gravitational decoupling, Einstein's equations

Introduction

A recent paper (Grib, Pavlov 2022) showed the possibility of phase transition near the event horizon of a black hole. However, the critical Hawking temperature (Hawking 1975) near the horizon at which the phase transition can happen is reached in the vicinity of the event horizonm, while the radius of the region, where this effect is possible, is negligible. When one considers particle collisions, the situation becomes better because the center-of-mass energy can grow unboundly in some processes (Banados et al. 2009; Grib, Pavlov 2015; Vertogradov 2022; Zaslavskii 2012). The region near the event horizon where phase transition can happen is much bigger in comparison to the Hawking temperature (Grib, Pavlov 2022). However, one can try to increase the effect caused by the Hawking temperature by considering the modification of the standard black hole solution and how these modifications affect the black hole temperature.

A well-known theorem in the black hole theory states that a black hole does not have hairs, i. e., it can have only three charges—a mass M, angular momentum j, and electric charge Q. However, it was shown that a black hole can have soft hair (Hawking et al. 2016). Recently, it was understood that one can obtain a hairy black hole by using the gravitational decoupling method (Contreras et al. 2021; Ovalle 2017). In most cases, obtaining new analytical solutions of the Einstein equations is an extremely hard task. One can solve these equations, for example, for the spherical symmetry and the perfect fluid as a source of gravitation. However, if we consider a more realistic case when the perfect fluid is coupled to another matter, it is nearly impossible to obtain the analytical solution. The gravitational decoupling through minimal geometrical deformation shows the possibility of decoupling two gravitational sources. One can write the energy-momentum tensor as:

$$T_{ik} = \tilde{T}_{ik} + \alpha \Theta_{ik} \ , \tag{1}$$

where $\tilde{T}_{ik}$ is the energy-momentum tensor of the perfect fluid and α is the coupling constant to the energy-momentum tensor Θ_{ik}. It is possible to solve Einstein's field equations for a gravitational source whose energy-momentum tensor is expressed as (1) by solving Einstein's field equations for each component $\tilde{T}_{ik}$ and Θ_{ik} separately. Then, by a straightforward superposition of the two solutions, we obtain the complete solution corresponding to the source T_{ik}. Since the Einstein equations are not linear, this method is effective for the analysis of the solution. It is an especially important tool when one faces the realistic cases, i. e., the stars and collapsing objects whose interior matter is far from the ideal perfect fluid.

By applying this method, a new modification of the Schwarzschild black hole was obtained (Ovalle et al 2018; 2021). These black hole solutions satisfy the strong and dominant energy conditions in the whole region from the event horizon up to infinity. All these solutions have been obtained by small deformations of the Schwarzschild vacuum. The hairy analogy of the Kerr spacetime has been obtained in (Mahapatra, Banerjee 2023). A more realistic case, i. e., the deformation of dynamical background was obtained in (Vertogradov, Misyura 2022).

In this paper, we consider the Hawking temperature for a hairy charged Reissner-Nordstrom black hole obtained by gravitational decoupling through minimal geometrical deformation. We also discuss how it deviates from the black hole when the hairs are ignored. All these models represent a black hole supported by a non-linear electrodynamics.

This paper is organized as follows: in Section 2 we briefly discuss two methods of obtaining the Hawking temperature for a general spherically-symmetric black hole. In Section 3 we explicitly calculate the Hawking temperature for the Reissner-Nordstrom black hole. In Section 4 we consider three models of a hairy black hole and their temperature and compare these results with a no-hair solution. Section 5 provides conclusions and discusses further research. The system of units $c = G = 1$ will be used throughout the paper. Also, we shall adopt the signature − , + , + , +.

Black hole thermodynamics

In this section we review the basic concepts related to the black hole thermodynamics. For more useful and thorough discussion on this subject, see, for example, the review in (Carlip 2009). We consider a spherically-symmetric line element in the form:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2 \ , \tag{2}$$

where a lapse function $f = f(r)$ depends upon radial coordinate r, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric of a unit two-sphere. To describe black holes, it is convenient to write (2) in the form:

$$ds^2 = -\left(1 - \frac{b(r)}{r}\right) dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad . \tag{3}$$

Here, we can refer to the function $b(r)$ as the shape function which specifies the shape of the spatial slice. In the limit $\lim\limits_{r \to infty} b(r)$ the shape function can be interpreted as asymptotic mass $2M$. We assume the asymptotic flatness for all models considered in this paper. Metric (3) has horizons at $b(r_h) = r_h$. This equation might have several roots but only the outermost horizon is of the major interest and we will consider only this one. We are interested in the case when $\forall r > r_h \to b(r) < r$ and $\frac{db(r)}{dr}|_{r=r_h} < 1$. The case $\frac{b(r)}{dr}|_{r=r_h} = 1$ corresponds to an extremal black hole for which the Hawking temperature is zero (Visser 1992). This case will not be considered within this paper. The Hawking temperature is given by:

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$$k_b T_h = \frac{\hbar \varkappa}{2\pi} \,, \tag{4}$$

where k_b is the Boltzmann constant, T_h is the Hawking temperature and $\varkappa$ is the surface gravity which, for metric (2), has the form:

$$\varkappa = \lim_{r \to r_h} \frac{1}{2} \frac{df}{dr} \,. \tag{5}$$

Substituting a lapse function in form (3), one obtains:

$$\varkappa = \frac{1}{2r_h}\left[1 - b'(r_h)\right] , \tag{6}$$

where a dash corresponds to the derivative with respect to the radial coordinate r.

Another way to obtain the Hawking temperature is to use Euclidean signature techniques. By the formal transformation to the imaginary time $t \to it$, we get:

$$ds^2 = f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2 \,. \tag{7}$$

Again, we are interested in the outermost horizon at $r = r_h$ and discard the whole $r < r_h$ region. Furthermore, we assume the lapse function f in the form given by (3) and also demand that a black hole is not an extremal one. Taylor expand near the horizon gives:

$$1 - \frac{b(r)}{r} \approx \left[1 - b'(r_h)\right] \frac{r - r_h}{r_h} \,. \tag{8}$$

Substituting (8) into (7), one obtains:

$$ds^2 \approx \frac{\left(1 - b'(r_h)\right)\left(r - r_h\right)}{r_h} dt^2 + \left[\frac{\left(1 - b'(r_h)\right)\left(r - r_h\right)}{r_h}\right]^{-1} dr^2 + r_h^2 d\Omega^2 \,. \tag{9}$$

By introducing a new variable R:

$$dR = \left[\frac{\left(1 - b'(r_h)\right)\left(r - r_h\right)}{r_h}\right]^{-\frac{1}{2}} dr \,, \tag{10}$$

one can transform metric (9) to obtain:

$$ds^2 \approx \frac{\left[1 - b'(r_h)\right]^2}{4r_h^2} R^2 dt^2 + dR^2 + r_h^2 d\Omega^2 \,. \tag{11}$$

The (t, R) part of this metric is similar to a flat two-plane in polar coordinates, with imaginary time t serving as the angular coordinate. In order to avoid a conical singularity, one should demand that $\frac{1 - b'(r_h)}{2r_h} t$ has a period 2π, i. e., t has a period β which is given by:

$$\beta = \frac{4\pi r_h}{1 - b'(r_h)} \,. \tag{12}$$

According to (Gibbons, Hawking 1977), this imaginary time t is interpreted as the existence of the thermal bath of a temperature $k_b T_h = \frac{\hbar}{\beta}$, which explicitly gives:

$$k_b T_h = \frac{\hbar}{4\pi r_h}\left(1 - b'(r_h)\right), \tag{13}$$

which coincides with (4).

The Hawking temperature of the Reissner-Nordstrom black hole

In this section we will apply the method described in the previous section to the Reissner-Nordstrom solution which describes a charged static black hole. The result of this section is well-known and can be found, for example, in (Brown et al. 1994; Poisson 2007; Visser 1992). We rederive these results only in order to compare them with hairy black holes.

Here and in what follows, we will use the system of units $k_b = \hbar = 1$. The lapse function f (2) in the Reissner-Nordstrom case is given by:

$$f(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2}. \tag{14}$$

The shape function $b(r)$ (3) and its derivative are given by:

$$b(r) = 2M - \frac{Q^2}{r}, \qquad b'(r) = \frac{Q^2}{r^2}. \tag{15}$$

The Reissner-Nordstrom black hole has two horizons which are located at:

$$r_\pm = M \pm \sqrt{M^2 - Q^2}. \tag{16}$$

Here, as we stated above, we are interested only in the outermost horizon, so that $r_h = r_+$. The cases $Q^2 > M^2$ and $Q^2 = M^2$, which correspond to a naked singularity and an extremal Reissner-Nordstrom black hole, will not be considered in this paper.

The surface gravity $\varkappa$ at the horizon is given by:

$$\varkappa = \frac{1}{r_h}\left(1 - \frac{Q^2}{r_h^2}\right). \tag{17}$$

And the Hawking temperature is:

$$T_h = \frac{1}{4\pi r_h}\left(1 - \frac{Q^2}{r_h^2}\right). \tag{18}$$

Note, that in the extremal case $M^2 = Q^2$, the horizon is located at $r_h = M$, which gives $1 - \frac{Q^2}{r_h^2} = 0 \rightarrow T_h = 0$.

Thermodynamics of hairy black holes

Using the gravitational decoupling method, a recent paper (Ovalle et al. 2021) introduced a new solution which describes the exterior geometry of hairy black holes. In this section, we will calculate the Hawking temperature and compare the results with a usual Reissner-Nordstrom black hole in order to find out how primary hairs affect the Hawking temperature.

Model 1

This model can be interpreted as a black hole supported by a non-linear electrodynamics. The other two models can be obtained from this model by defining the electric charge Q as a function of the Schwarzschild mass M and primary hairs α and $l_0 = \alpha l$.

The lapse function f for Model 1 is given by:

$$f(r) = 1 - \frac{2\mu}{r} + \frac{Q^2}{r^2} - \frac{\alpha(\mu - l_0/2)e^{-r/(\mu - l_0/2)}}{r}. \tag{19}$$

Here, $\mu = M + l_0/2$, M is the Schwarzschild mass, α is the coupling constant and $l_0 = \alpha l$ is the primary hair, Q can be interpreted as the electric charge of a black hole. The influence of the geodesic motion of primary hairs was studied in (Ramos et al. 2021). The influence of these parameters for the thermodynamics properties in a hairy Schwarzschild black hole was investigated in (Cavalcanti et al. 2022). The shadow properties of hairy black hole are covered in (Vagnozzi et al. 2022). The Schwarzschild solution is the limit of $\alpha \to 0$.

The event horizon equation is given by $f(r_h) = 0$, which can be solved with respect to l_0 to give:

$$l_0 = r_H - 2M + \frac{Q^2}{r_H} - \alpha M e^{-r_H/M} \ . \tag{20}$$

We have several restrictions on the parameters. First of all, one should realize that like in the pure Reissner-Nordstrom case, one should impose certain conditions on the parameters in order to avoid a naked singularity in the usual Reissner-Nordstrom case. They include M and charge Q - $M^2 \geq Q^2$. However, the condition $M^2 = Q^2$ is not forbidden because in this case the hairy black hole (19) is not an extremal one. Also, this model satisfies the dominant energy condition only when $r \geq 2M$, so we do not consider the region $0 \leq r \leq 2M$ and demand $r_h \geq 2M$ to satisfy the energy condition. The fulfilling of the dominant energy condition also imposes the following restrictions on the parameters Q and l:

$$|Q| \geq \alpha \frac{M^2}{e^2}, \qquad l \geq \frac{M}{e^2} \ . \tag{21}$$

The Hawking temperature (4) $T_h = \dfrac{\varkappa}{4\pi}$ is given in terms of the surface gravity $\varkappa$ which for this model reads:

$$2\varkappa = 1 + \frac{2M}{r_h^2} - \frac{2Q^2}{r_h^3} + \frac{\alpha r_h e^{-r_h/(M - l_0/2)} + \alpha\left(M - l_0/2\right) e^{-r_h/(M - l_0/2)}}{r_h^2} \ . \tag{22}$$

So the Hawking temperature is:

$$T_h = \frac{1}{4\pi} 1 + \frac{2M}{r_h^2} - \frac{2Q^2}{r_h^3} + \frac{\alpha r_h e^{-r_h/(M - l_0/2)} + \alpha\left(M - l_0/2\right) e^{-r_h/(M - l_0/2)}}{r_h^2} \ . \tag{23}$$

Fig.1 is plotted for $M = 1$, $\alpha = 0.5$, $Q = 0.9$. This figure shows the dependence of the primary hair l_0 on the horizon location (a blue curve). The horizontal red line corresponds to $l_0 = \alpha \dfrac{M}{e^2}$. We see that at $r_h \geq 2.061$ the dominant energy condition is always held.

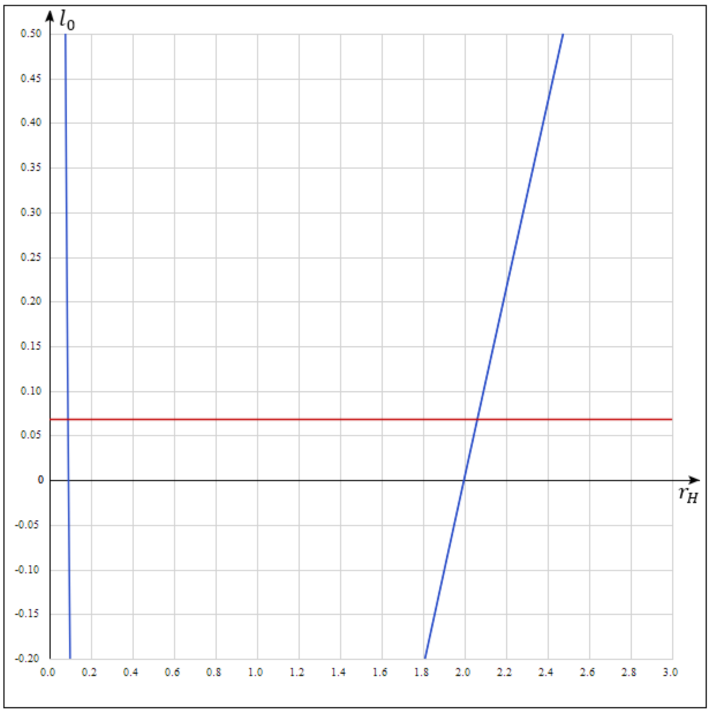

Fig. 1. The event horizon function

Fig. 2 shows the dependence of the Hawking temperature T_h on the horizon location. Three horizontal lines correspond to the Hawking temperature of a usual Reissner-Nordstrom black hole. We have picked up the following parameters for horizontal lines: $M = 1$, $Q = 0.5$ (a red line); $Q = 0.9$ (a green line) and $Q = 0.99$ (an orange line). In this model, the Hawking temperature of a hairy black hole does not depend on the electric charge Q and is the function only of mass M, coupling constant $\alpha = 0.5$ and horizon location r_h, i. e., $T_h \equiv T_h(M, \alpha, r_h)$ (the corresponding curve is blue). We can easily see that at $r_h \in (2.061, 0.212)$ for $Q = 0.$; $r_h \in (2.061, 2.481)$ $r_h \in (2.061, 2.481)$ for $Q = 0.9$ and $r_h \in (2.061, 4.583)$ for $Q = 0.99$, the Hawking temperature is higher than for a usual Reissner-Nordstrom black hole. It means that in these cases the phase transition, which can happen near the event horizon, can be fairer than in a no-hair black hole. One can also see that when one considers an extremal Reissner-Nordstrom black hole, then the Hawking temperature is absent but in the hairy case it is not zero. So, we can conclude that for the first model the primary hairs can increase the Hawking temperature in comparison with the usual Reissner-Nordstrom case.

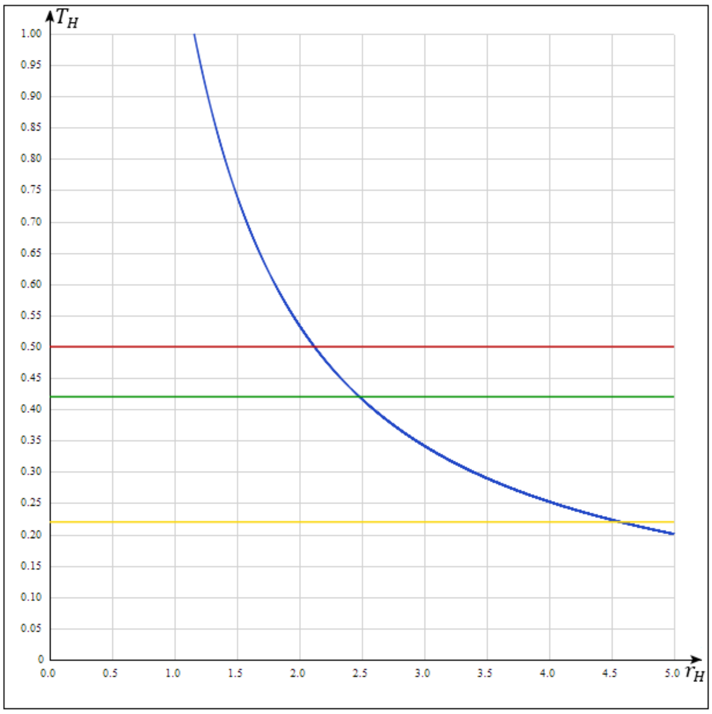

Fig. 2. The Hawking temperature. Model 1

Model 2

As we stated in the previous subsection, the ensuing models differ from the first one by the appropriate choice of the function Q. In this model Q is given by:

$$Q^2 = L_0 M\left(2 + \alpha e^{-l_0/M}\right). \tag{24}$$

Substituting this into (19), one obtains:

$$f = 1 - \frac{2M + l_0}{r} + \frac{2l_0 M}{r^2} - \frac{\alpha M e^{-r/M}}{r^2}\left(r - l_0 e^{\frac{r-l_0}{M}}\right). \tag{25}$$

For this choice of the charge function the horizon equation is:

$$r_h = l_0 . \tag{26}$$

Like in the previous subsection, the Hawking temperature is given in terms of the first derivative of the lapse function f', which in this case reads:

$$f' = \frac{2M + l_0}{r_h^2} - \frac{4l_0 M}{r_h^3} + \frac{\alpha e^{-r_h/M}\left(r_h - 2M\right)}{r_h^3}\left(r_h - l_0 e^{\frac{r_h - l_0}{M}}\right) + \\ + \frac{\alpha M e^{-r_h/M}}{r_h^2}\left(\frac{l_0}{M} e^{\frac{r_h - l_0}{M}} - 1\right). \tag{27}$$

In this model, the dominant energy condition is always held for $r \geq 2$. Again, like in the first model, the Hawking temperature, after substituting l_0, does not depend on the electric charge of a hairy black hole.

Fig. 3 shows how the Hawking temperature T_H depends on the event horizon location r_h. The choice of the parameters is like in the first model. Fig. 3 differs from Fig. 2 only by the blue curve, which corresponds to the Hawking temperature of a hairy black hole. From the figure, one can see that the temperature of a hairy black hole is always less than in a no-hair case. The only exception is an extremal Reissner-Nordstrom black hole. So we can conclude that one can consider a bigger region where phase transition can happen only for an extremal Reissner-Nordstrom black hole. In other cases, the region is smaller than in a no-hair case.

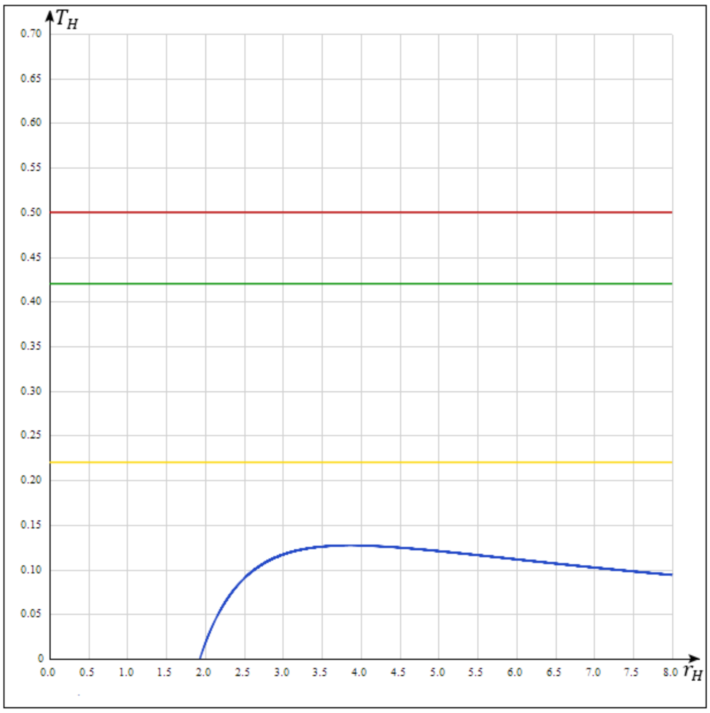

Fig. 3. The Hawking temperature. Model 2

Model 3

For the last model, which we consider within this paper, the charge function is given by:

$$Q^2 = \alpha M\left(2M + l_0\right)e^{-\frac{2M + l_0}{M}}. \tag{28}$$

Its substitution into the lapse function f (19), gives us:

$$f = 1 - \frac{2M + l_0}{r} - \frac{\alpha M}{r^2} e^{-r/M}\left(r - \left(2M + l_0\right)e^{\frac{r - 2M - l_0}{M}}\right). \tag{29}$$

The horizon in this model is located at:

$$r_H = 2M + l_0. \tag{30}$$

The first derivative of the lapse function, which gives the main contribution to the Hawking temperature, reads:

$$f' = \frac{2M + l_0}{r_h^2} + \frac{2\alpha M}{r_h^3} e^{-r_h/M} \left(r_h - (2M + l_0) e^{\frac{r_h - 2M - l_0}{M}} \right) + \\ + \frac{\alpha}{r_h^2} e^{-\frac{r_h}{M}} (r_h - 2M + l_0) e^{\frac{r_h - 2M - l_0}{M}} - \frac{\alpha M}{r_h^2} e^{-\frac{r_h}{M}} \left(1 - (2 + l_0 / M) e^{\frac{r_h - 2M + l_0}{M}} \right). \quad (31)$$

In this model, the dominant energy condition is held when $r_h > 2.067$. It should not be a surprise that for this model the Hawking temperature does not depend on the electric charge of a hairy black hole again.

Fig. 4 is plotted for the same parameters as in the two previous cases and shows the Hawking temperature T_H as the function of the event horizon location r_h. From the figure, one can see that the hairy black hole temperature at $r_h \in (2.067, 2.074$ for $Q = 0.5$; $r_h \in (2.067, 2.444)$ for $Q = 0.9$ and $r_h \in (2.067, 4.583)$ for $Q = 0.99$, respectively, is bigger than in a no-hair Reissner-Nordstrom black hole. This model shows that under the proper choice of parameters, the region where the phase transition can take place is bigger than in a usual charged black hole.

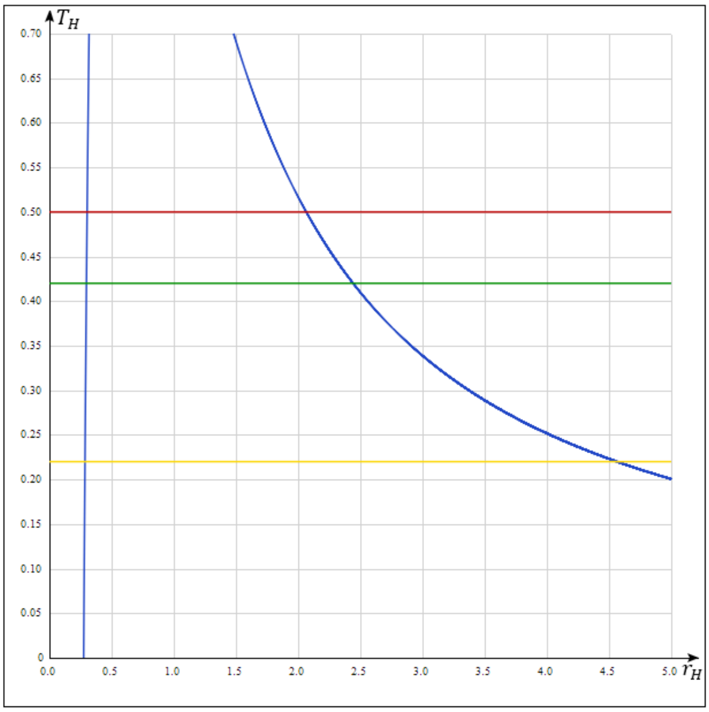

Fig. 4. The Hawking temperature. Model 3

Conclusions

In this paper, we have derived the Hawking temperature of the hairy charged black holes by gravitational decoupling. These black holes can be interpreted as ones supported by a non-linear electrodynamics. The main analytical result obtained within this paper is that the Hawking temperature does not depend on the electric charge Q of a hairy black hole.

We have considered three models and showed that the Hawking temperature is not zero in the case of an extremal no-hair Reissner-Nordstrom black hole. However, in the second model, the Hawking temperature of the hairy charged black hole is always less than one in the case of a non-extremal no-hair charged black hole. For the first and third models, we showed that under a certain choice of primary hairs, one can obtain the black hole temperature higher than in a non-extremal no-hair Reissner-Nordstrom

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black hole. It means that the region, where the phase transition can take place in the first and the third models, is bigger than the one in a usual charged black hole.

Further research will focus on the entropy, heat capacity and stability of the hairy black hole. It will also identify the exact value of r at which the phase transition can take place. Besides, our follow-up papers will investigate particle collisions, corresponding temperature and how hairs of a black hole affect collision.

Conflict of Interest

The authors declare that there is no conflict of interest, either existing or potential.

Authors contribution

The authors have made an equal contribution to the preparation of the paper.

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