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Wind power variations in frequency domain: Introducing a concept of fractional dynamics

A. F. Belo ¹, K. Shimakawa ²

¹ East Timor National University, Av. Cidade de Lisboa, Díli, East Timor

² Gifu University, 1–1 Yanagido, Gifu 501-1193, Japan

Authors

Abelito Filipe Belo, ORCID: [0000-0003-1759-9470](https://orcid.org/0000-0003-1759-9470), e-mail: abelitofilipe@gmail.com

Koichi Shimakawa, e-mail: koichi@gifu-u.ac.jp

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Abstract. We apply a new concept of fractional dynamics in the equation of motion, specifically the harmonic oscillator in non-Euclidean space, to analyze wind speed (WS) in the frequency domain. Frequency spectra (FS) for WS are expressed well by the Mittag-Leffler function (MLF) originating from a concept of *fractional dynamics* in complex systems: FS behaves as $1/f^\alpha$ in high frequencies ($\alpha < 1$). All these findings are compared with those for air temperature (AT), showing $1/f^\alpha$ for WS and $1/f^{2\alpha}$ for AT. An interesting relationship between the $1/f^\alpha$ -law found for WS and $1/f^{2\alpha}$ for AT will be discussed in terms of Newtonian dynamics in the fractional space.

Keywords: wind power, Fast Fourier transform (FFT), fractional relaxation and dynamics, Mittag-Leffler function (MLF), non-Euclidean space, Newtonian dynamics

Introduction

Wind power, one of the major inputs for producing sustainable environmental conditions on the globe (Hasche 2010; Joselin Herbert et al. 2007), is now increasing significantly, while the generated power itself is not stable over time. There are several ways to analyze wind characteristics in the time and/or frequency domain. In the time domain, a simple way is to statistically construct a histogram of the step size (wind speed, etc.) in output over time (Murthy, Rahi 2014; Wan, Bucaneg 2002), and/or variogram time-series analysis has been proposed (Liu et al. 2016). To better understand the nature of *wind quality*, the frequency spectrum (FS) in the frequency domain has been discussed in addition to the time domain, which facilitates the understanding of detailed characteristic features involving periodicity, etc. (Apt 2007).

This paper will identify characteristic features in WS in the frequency domains in three different locations: the Hera campus, Universidade Nacional Timor Lorosa'e, East Timor; the Takayama area (near the mountainside), Gifu, Japan; and the Yanagido campus at Gifu University (Gifu, Japan). We will also discuss ambient temperature (AT) variations in the frequency domains in the same locations.

The WS variations are modeled by the equation of motions, i. e., Newtonian or Lorentz dynamics, as a *fractional* harmonic oscillator, which will be useful in understanding the dynamics in so-called complex systems (Metzler, Klafter 2000). It will be found that the FS behaves as $1/f^\alpha$ for the WS

and $1/f^{2\alpha}$ for the AT ($\alpha < 1$) in high frequencies, while the FS takes a constant in lower frequencies, supporting a significant role of the Mittag-Leffler function introduced in complex systems. An interesting relationship between the $1/f^\alpha$ -law found for WS and $1/f^{2\alpha}$ for AT will also be discussed within a category of Newtonian dynamics.

Measurement system and analysis

First, we discuss WS at the Hera campus, located 10 km east of the Díli city (East Timor) (8.55° S latitude, 125.56° E longitude). The data analyzed in this work were collected from June to September 2019. A 600-watt wind generator set (Nantong R&X Energy Technology: RX-600H3) was utilized. WS was measured at 6-m instrumented towers. The output voltage of the wind turbine was transferred to a data logger (Belo, Shimakawa 2021). Data for WS at every 100 seconds were stored in a PC.

The second example is WS and AT at two stations at the Takayama Field Station, and the third — at the Yanagido campus at Gifu University. The meteorological data observed at the Takayama Field Station of the River Basin Research Center, Gifu University, is used in this study. The Takayama Field Station is located at an elevation of 1.342 m on a high mountainside at 36°8' N latitude and 137°26' E longitude in Gifu, Japan. The observation site is a basin in a complex topography, and the observation instrument is located in an open space of about 1,100 m² surrounded by evergreen coniferous trees of about 20-m height. The wind and temperature sensors are a propeller anemometer (Young 05103-47) and a resistive platinum one (Vaisala HMP-45D), located at a height of 4.23 m and 2.74 m, respectively, from the ground. These data are sampled instantaneously every 10 minutes. The period of the observation data used in this study is 10 years, from January 2008 to December 2017. Another location with the same system is the Yanagido campus, Gifu University (35.4168° N latitude and 136.7769° E longitude). The data are also sampled instantaneously every 10 minutes. The period of the observation data used in this study is 6 years, from January 2017 to December 2022.

All the data of wind speed $v(t)$ are Fourier transformed:

$$v(f) = \int_{-\infty}^{\infty} v(t) e^{-i2\pi ft} dt, \quad (1)$$

where f is the frequency (s⁻¹) and $v(f)$ is the frequency spectrum (FS) (Kogan 2008; Papoulis 1962).

We took the MATLAB program for the Fast Fourier Transform (FFT). When the sampling time is Δt and the data number is given by N , the frequency interval is given as $\Delta f = 1/N\Delta t = 1/t_p$, where t_p is the period of measurement time. The minimum frequency in the FS (by FFT) is therefore $1/t_p$, and the maximum is set to $1/2\Delta t$ (Kogan 2008; Papoulis 1962).

Fractional relaxation and dynamics

Let us begin with the dynamic motion of the particles, with displacement x having a mass of M . It is called the equation of motion, which is the *most fundamental dynamic equation* (Newtonian), given as (Metzler, Klafter 2000)

$$M \frac{d^2 x}{dt^2} + M\Gamma \frac{dx}{dt} + M\omega_0^2 x = F, \quad (2)$$

where Γ is the damping rate, ω_0 is the natural frequency, and F is the external force in Newtonian classical mechanics. A popular example of equation (2) is the so-called harmonic oscillator (HO) (in Euclidean space). The last two terms in equation (2) exhibit the *relaxation* process:

$$M\Gamma \frac{dx}{dt} + M\omega_0^2 x = \frac{dx}{dt} + \frac{x}{\tau} = \frac{F}{M}, \quad (3)$$

where $\omega_0^2/\Gamma \equiv 1/\tau$.

Let us now introduce the concept of fractional dynamics into equation (3). The α -th derivative equation ($0 < \alpha < 1$) is basically used for the fractional relaxation process (in non-Euclidean space) (Belo et al. 2024; Metzler, Klafter 2000).

$$\frac{d^\alpha x}{dt^\alpha} = -\frac{x}{\tau^\alpha}, \quad (4)$$

where F is applied at $t = 0$ and τ^α produces dimensional correctness. The solution of the above equation is given by

$$x(t) = E_\alpha\left(-\left(t/\tau\right)^\alpha\right) = \sum_{k=0}^{\infty} (-1)^k \frac{(t/\tau)^k}{\Gamma(\alpha k + 1)}, \quad (5)$$

which is called the Mittag-Leffler function (Belo et al. 2024; Metzler, Klafter 2000). When $\alpha = 1$, $x(t)$ gives just the exponential function, $\exp(-t/\tau)$. The MLF in the frequency domain is obtained easily by the Laplace transform (LT) of equation (4):

$$s^\alpha x(s) - s^{\alpha-1} x(0) = -\tau^{-\alpha} x(s), \quad (6)$$

where $x(0)$ is the initial value, and s is the Laplace frequency given by $i\omega$. Then we get $x(s)$ as

$$x(s) = \frac{s^{\alpha-1} x(0)}{s^\alpha + \tau^{-\alpha}} = \frac{x(0)}{s(1 + (s\tau)^{-\alpha})}. \quad (7)$$

As $x(s)$ is a position function, the wind speed can be obtained from its derivative. As the MLF is a decay function with t , $v(t)$ is therefore given as $-df(t)/dt$ (Rosa, de Oliveira 2015); the Laplace transform of the above equation gives

$$v(s) = -C(sx(s) - x(0)) = C(1 - sx(s)) = C\left(1 - \frac{s}{s(1 + (s\tau)^{-\alpha})}\right) = \frac{C}{1 + (s\tau)^\alpha}, \quad (8)$$

where C is a constant, $x(0)$ is set to 1, and τ is a relaxation time. Equation (8) is mathematically equivalent to the Cole–Cole relation in dielectrics (solid-state physics) (Cole, Cole 1941), showing a characteristic low-pass filter.

Results and discussion

Fig. 1 shows the measured time-dependent wind speed $v(t)$ at every 100 seconds for three months (8×10^6 s) at the Hera campus (June–September 2019). The maximum wind speed is 35 m/s, and the minimum is 0 m/s, with roughly 1-day periodicity (diurnal), which is a general trend in wind power reported in other areas (Apt 2007).

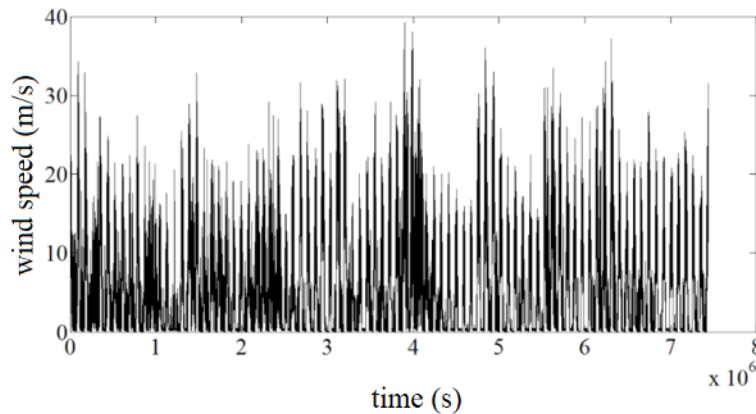


Fig. 1. Time-variation of the wind speed $v(t)$ at every 100 seconds for three months (8×10^6 s) at the Hera campus (June–September 2019)

Let us discuss the wind speed variation in the frequency domain. Fig. 2 shows the frequency spectrum (FS: black solid line), $|v(f)|$, directly calculated from the FFT of $v(t)$, together with a model prediction (red solid line) given by equation (8), with $C = 0.50$, $\alpha = 0.50$, and $\tau = 1.0 \times 10^5$ s. At around 10^{-5} Hz,

a peak is observed, which corresponds to a 1-day period. The 2nd harmonicity (peak) appears at a half-day cycle. Fitting equation (8) to practical data is reasonably good.

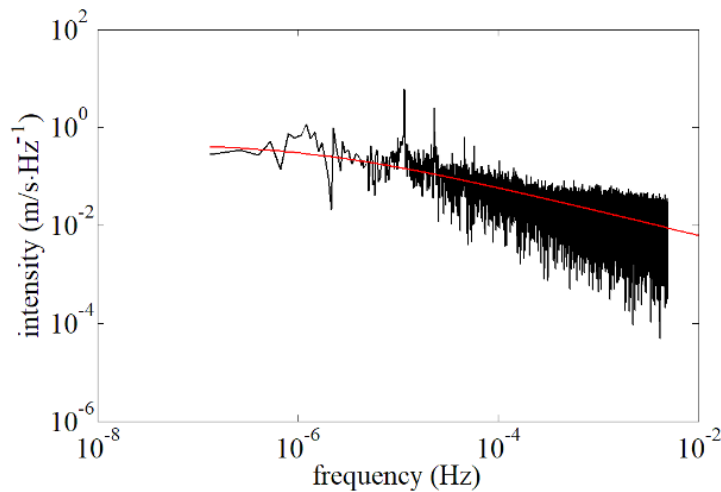


Fig. 2. Frequency spectrum $|\nu(f)|$ of wind speed at the Hera campus. The solid line is predicted from equation (8), with $C = 0.50$, $\alpha = 0.50$, and $\tau = 1.0 \times 10^5$ s ($R^2 = 0.41$)

Let us discuss the WS variation in other locations. Fig. 3 shows the measured time-dependent WS at every 600 seconds for 10 years (from January 2008 to December 2017: about 3.2×10^8 s) at the Takayama Field Station. The maximum WS is 5 m/s, which is observed around August, and the minimum is around 0 m/s. The maximum wind speed is small because strong wind is shaded by the complex topography and tall trees around the observation site. The WS time series shows the periodicity in a one-year period (Climate change monitoring report 2015).

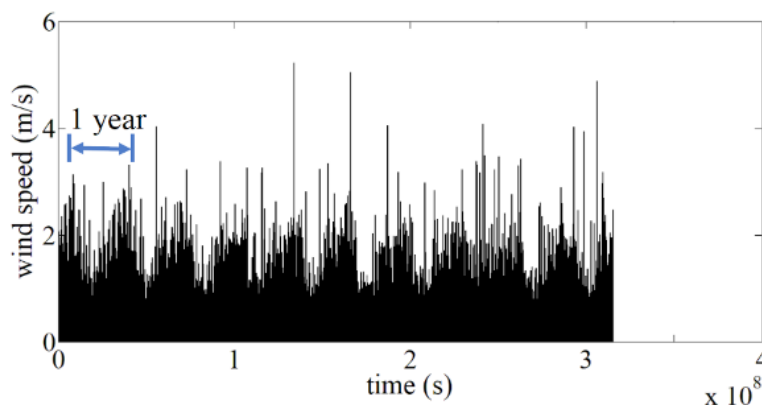


Fig. 3. Time domains of wind speed from January 2008 to December 2017 at Takayama

Fig. 4 shows the FS for the WS. The FS shows the principal peaks around 1 year and 1 day. Around a 1-year peak, it takes a constant value and decreases with increasing frequency. The solid (red) line shows the applicability of fractional dynamics prediction from equation (8) with $C = 0.01$, $\alpha = 0.60$, and $\tau = 2 \times 10^5$ s for the WS with $R^2 = 0.46$.

Fig. 5 shows the measured time-dependent WS at every 600 seconds for 6 years (from January 2017 to December 2022: about 2.1×10^8 s) at Yanagido as the last example. The maximum WS is ~ 20 m/s, observed in or around August.

Fig. 6 shows the FS for the WS, estimated from the raw time-dependent data indicated in Fig. 5. Overall features are almost the same as those in Fig. 4. The solid (red) line in (a) denotes the applicability of fractional dynamics prediction from equation (8) with $C = 0.07$, $\alpha = 0.6$, and $\tau = 2.0 \times 10^5$ s for the WS with $R^2 = 0.53$, in which α is also a bit larger than that ($= 0.50$) of the Hera campus (see Fig. (2)).

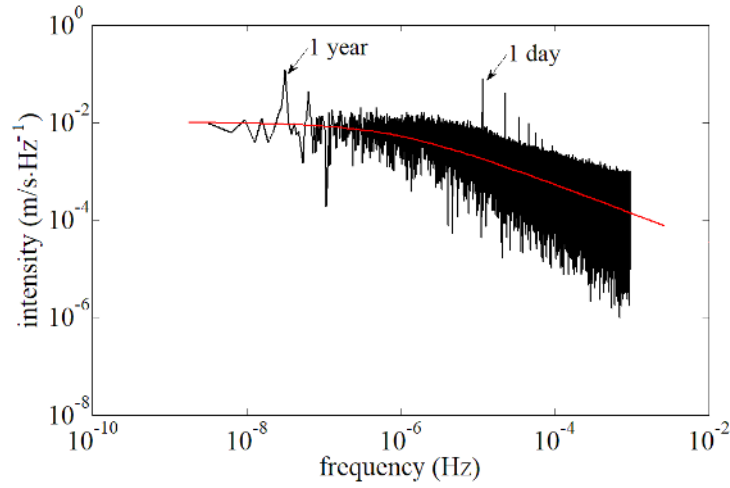


Fig. 4. Frequency spectrum $|F(f)|$ of WS at Takayama for 2008 to 2017, together with the model prediction (red solid line), with $C = 0.01$, $\alpha = 0.60$, and $\tau = 2.0 \times 10^5$ s ($R^2 = 0.46$)

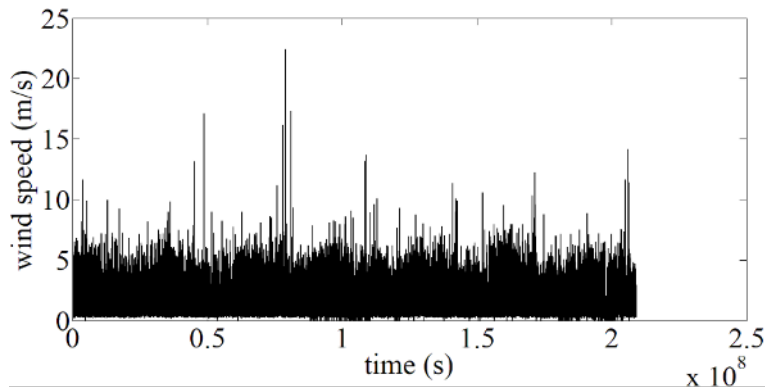


Fig. 5. Time domains of wind speed from January 2017 to December 2022 at Yanagido (Gifu)

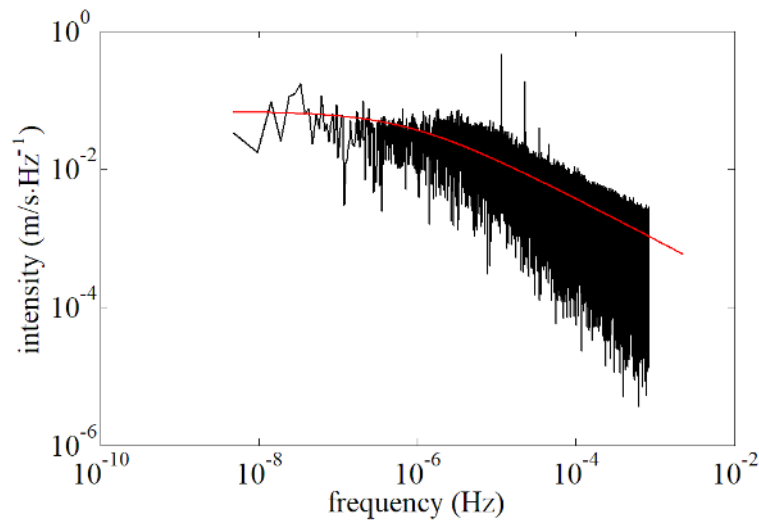


Fig. 6. Frequency spectrum $|F(f)|$ of WS at Yanagido for January 2017 to December 2022, together with the model prediction (red solid line), with $C = 0.07$, $\alpha = 0.60$, and $\tau = 2.0 \times 10^5$ s ($R^2 = 0.53$)

Fig. 7 presents the atmospheric (air) temperature variations measured at Takayama (a) and the Yanagido campus, Gifu University (b). The data were collected every 600 seconds for 10 years ($\sim 3.2 \times 10^8$ s) from January 2008 to December 2017 at Takayama and for 6 years ($\sim 2.1 \times 10^8$ s) from January 2017 to December 2022 at the Yanagido campus. The maximum AT is 34°C and 38°C ($\sim$ in August), and the minimum is -15°C and -5°C ($\sim$ in February), respectively. It means that AT at Takayama is much colder than that at the Yanagido campus. The AT time series shows the periodicity in one year for both. In wintertime (December–January–February), northwesterly winds cause the advection of cold air from Siberia to Japan, bringing heavy snowfall to Japan's Sea of Japan side. At this time the temperature tends to be low (Japan Meteorological Agency 2015). In summertime (June–July–August), in the second half of summer, the North Pacific High extends northwestwards around Japan, bringing hot and sunny conditions to the country. At this time the temperature tends to be high, causing cool and moist easterly winds (Japan Meteorological Agency 2015).

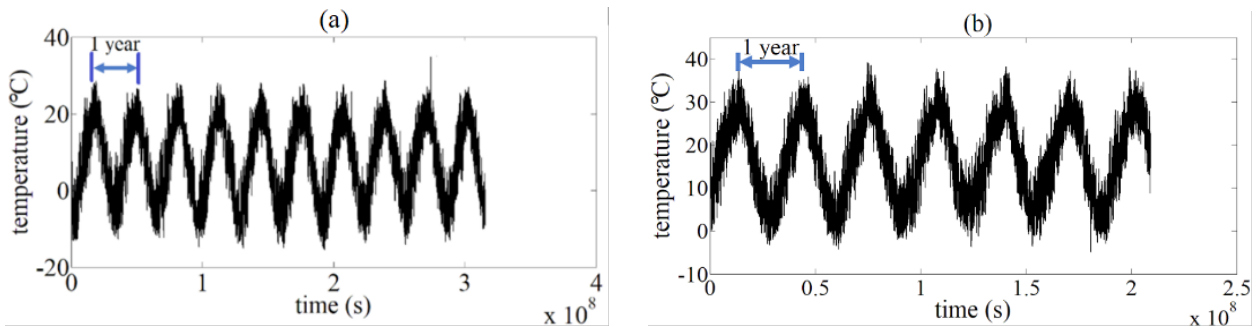


Fig. 7. Time domain AT (a) from January 2008 to December 2017 at Takayama and (b) from January 2017 to December 2022 at Yanagido (Gifu)

Fig. 8 shows frequency spectrum (FS) for the AT at (a) Takayama and (b) the Yanagido campus. FS shows the principal peaks around 1 year and 1 day. Around a 1-year peak, it takes a constant value and decreases with increasing frequency, which is the same behavior as the $|F(f)|$ of WS (see Figs. 4 and 7). We therefore try to use the same equation as equation (8). Note, however, that a constant B is used for AT instead of C in equation (8). The solid (red) line in (a) shows the prediction from equation (8) with $\alpha = 1.1$, $\tau = 2 \times 10^5$ s, and $B = 0.20$ ($R^2 = 0.56$), and in (b) we get $\alpha = 1.1$, $\tau = 2 \times 10^5$ s, and $B = 0.22$ ($R^2 = 0.58$). Note that the α -values for AT are almost twice those of WS, while τ is kept the same. As the kinetic energy of wind should be proportional to the squared wind speed v^2 , $T(f)$ is therefore proportional to $|v(f)|^2$. This is why the α -value for AT shows roughly twice that for WS.

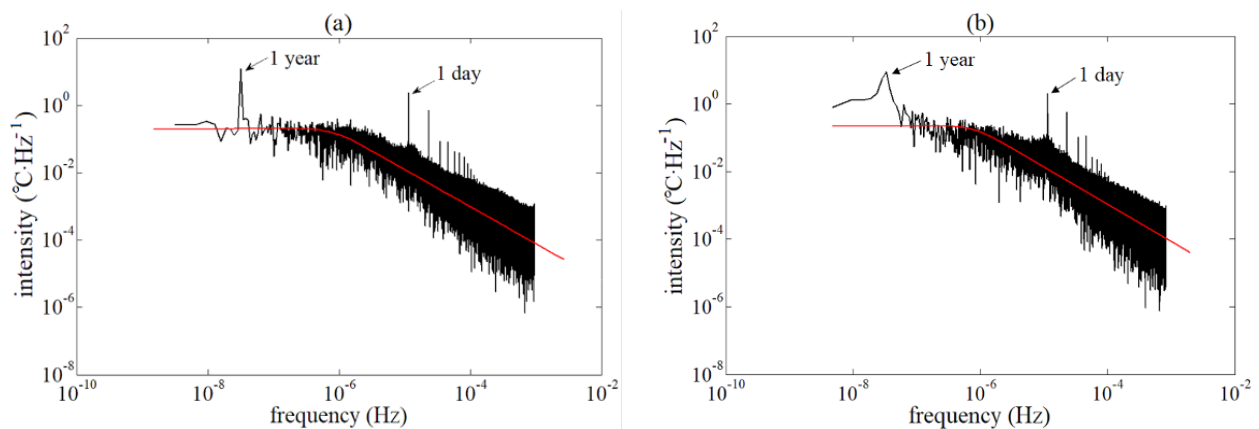


Fig. 8. Frequency spectrum $|T(f)|$ of air temperature at (a) Takayama for 2008 to 2017 and (b) Yanagido for January 2017 to December 2022. The solid line is given from equation (8), with $\alpha = 1.10$, $\tau = 2 \times 10^5$ s, and $B = 0.20$ ($R^2 = 0.56$) (a), and with $\alpha = 1.10$, $\tau = 2 \times 10^5$ s, and $B = 0.22$ ($R^2 = 0.58$) (b), respectively.

Conclusion

The variation of the wind speed and air temperature through time- and frequency-domains at different locations in East Timor and Japan was discussed by introducing a new concept of fractional dynamics. It was found that the frequency of wind speed (frequency histogram) is approximately given by the well-known Weibull distribution in all locations. We found that the dynamic change for WS is replicated well by the Mittag-Leffler function, originating from a concept of the *fractional dynamics* in complex systems. The FS behaves as $1/f^\alpha$ for the WS and $1/f^{2\alpha}$ for the AT in the high-frequency range ($\alpha < 1$). An interesting correlation between WS and AT was discussed in terms of Newtonian dynamics.

Conflict of Interest

The authors declare that there is no conflict of interest, either existing or potential.

Author Contributions

All authors made an equivalent contribution to the preparation of the publication.

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