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Beyond the standard quantum Zeno effect

A. A. Borisov ¹, A. A. Borisov ¹, T. F. Kamalov ¹

¹Federal State University of Education, 10A Radio Str., Moscow 105005, Russia

Authors

Andrey A. Borisov, ORCID: [0009-0004-3073-8505](https://orcid.org/0009-0004-3073-8505), e-mail: andreyborisov1998@mail.ru

Anton A. Borisov, e-mail: 0009-0004-7766-1322, e-mail: toni.borisov.98@mail.ru

Timur F. Kamalov, ORCID: [0000-0002-4349-4747](https://orcid.org/0000-0002-4349-4747), e-mail: timkamalov@mail.ru

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Abstract. In this paper, we introduce a functional for the decoherence rate of a quantum system. The quantum system is modeled as a qubit that can be in the state $|1\rangle$, $|0\rangle$, or in a superposition. In addition to external dephasing and the Zeno attenuation rate acting on the quantum system, resource losses of the measuring channel are taken into account. It is shown that for the parameters corresponding to the limit of ideally frequent measurements in the quantum Zeno effect, the stability of the qubit state is not increased. The existence of optimal values of the measurement time interval and the measurement imperfection parameter is established; at these values, the decoherence rate of the qubit state reaches a minimum. Calculations show that in the case under consideration, the qubit coherence time increases by a factor of 11.5, which ensures an increase in the stability of the qubit state.

Keywords: stability, qubit, quantum Zeno effect, minimum, measurement imperfection, stability principle

Introduction

Decoherence limits the efficiency of quantum systems by affecting the quantum state. One approach to increasing the stability of a quantum system (Sokolovski et al. 2012) is to use the quantum Zeno effect, which involves suppressing its evolution through a sequence of frequent ideal measurements. This effect was first theoretically predicted in (Khalfin 1968) and subsequently received experimental confirmation in various physical systems (Itano et al. 1991; Misra, Sudarshan 1977).

As part of the proposed model, which takes into account resource losses of the measuring channel, the functional of the total decoherence rate diverges in the limit of ideally frequent measurements. The stability principle (Kamalov 2025; 2026) is based on the requirement that the first variation of a functional is zero and the second variation is positive. This principle characterizes the stability of quantum-system states with respect to small perturbations.

The stability principle is applied here to the functional of the total decoherence rate, which acts as an action analogue for the process of measuring the state of a quantum system. The aim of this work is to analyze how the measurement imperfection parameter affects the stability of a quantum system against decoherence.

Model and functional of decoherence rate

Qubit: quantum Zeno effect and imperfect measurements

The quantum system under consideration is a qubit, which can be in the state $|1\rangle$, $|0\rangle$, or in a superposition. The Hamiltonian H describes the evolution of the quantum state of a qubit:

$$H = \frac{\hbar\Omega}{2} \sigma_x,$$

where Ω is Rabi frequency, and σ_x — Pauli matrix operator.

The evolution of the quantum state of a qubit over a measurement time interval τ is described by a unitary operator $U(\tau) = \exp(-iH\tau)$. We assume that at the initial moment of time, the qubit is in the state $|0\rangle$:

$$|\psi(0)\rangle = |0\rangle.$$

The state of the qubit $|\psi(\tau)\rangle$ over an interval τ at $\hbar = 1$ is determined as follows:

$$|\psi(\tau)\rangle = \cos \frac{\Omega\tau}{2} |0\rangle - i \sin \frac{\Omega\tau}{2} |1\rangle.$$

The probability of detecting $P_{|0\rangle}(\tau)$ a qubit in the initial state:

$$\begin{aligned} P_{|0\rangle}(\tau) &= |\langle 0|\psi(\tau)\rangle|^2 \\ P_{|0\rangle}(\tau) &= \cos^2 \frac{\Omega\tau}{2}. \end{aligned}$$

For infinitesimal functions:

$$\cos^2 \frac{\Omega\tau}{2} \cong 1 - \frac{\Omega^2\tau^2}{4} + O(\tau^4). \quad (1)$$

Let us write the expression for $P_{|0\rangle}(\tau)$ taking into account (1):

$$P_{|0\rangle}(\tau) \cong 1 - \frac{\Omega^2\tau^2}{4}.$$

Let $N = t/\tau$ consecutive measurements be performed during time t . Then the probability that the qubit will remain in the initial state is $|0\rangle$:

$$P_{|0\rangle}(\tau) \cong \left(1 - \frac{\Omega^2\tau^2}{4}\right)^N.$$

The expression for the probability of a qubit remaining in its initial state $P_{|0\rangle}(t)$ is defined as:

$$P_{|0\rangle}(t) \cong \exp\left(-\frac{\Omega^2\tau}{4}t\right).$$

Thus, the expression for the Zeno attenuation rate is written as:

$$\gamma_Z(\tau) \approx \frac{\Omega^2}{4} \tau.$$

We introduce the parameter $\lambda \in (0,1)$, which characterizes measurement imperfection. Physically, it reflects the limitations of the measurement setup (for example, the finite efficiency of the detector). As λ decreases, the efficiency of the measuring channel diminishes, which leads to an increase in the interval between successive measurements to a value τ/λ :

$$\gamma_z(\tau, \lambda) \approx \frac{\Omega^2 \tau}{4 \lambda}. \quad (2)$$

As $\lambda \rightarrow 1$ expression (2) tends to the standard quantum Zeno-effect form. This parameterization is not a universal law of quantum measurements and is used as a simple phenomenological model of the influence of limited resources of measuring devices on the dynamics of a quantum system. In addition to the Zeno slowing of the quantum system's state, the system is subject to external dephasing γ . Further analysis of the model under study compares the characteristic frequency Ω with external dephasing:

$$\Omega \approx \gamma. \quad (3)$$

Approximation (3) simplifies analytical and numerical estimates of the qubit state coherence time. The model assumes that qubit state measurements are accompanied by resource losses. These losses are introduced as an additional channel to qubit state decoherence, competing with both Zeno slowdown and external dephasing.

Resource losses of the measuring channel

To take into account the resource losses of the measuring channel with the measurement imperfection parameter, the following dependence is used:

$$\beta(\lambda) = \beta_0 \frac{1+\lambda}{(1-\lambda)^p},$$

where β_0 is a positive constant and $p > 0$ characterizes the rate of growth of losses in the measuring channel.

The proposed dependence is determined by the following factors:

- When the parameter λ decreases, the resource losses of the measuring channel tend to a certain value ($\beta(\lambda \rightarrow 0) \rightarrow \beta_0$);
- When the ideal projective measurement regime is approached, the resources of the measuring channel increase without the ($\beta(\lambda \rightarrow 1) \rightarrow \infty$) limit. This leads to an increase in the system's sensitivity to hardware limitations.

The chosen form of the $\beta(\lambda)$ dependence is a phenomenological coarse-grained description of measurement-channel losses and is consistent with the general conclusion of works (Deffner et al. 2016; Guryanova et al. 2020) about the unlimited growth of resource costs when approaching ideal measurements. It may describe, for example, finite detector efficiency, detector dead time, classical processing overhead, heating, and back-action costs that increase when the measurement becomes closer to an ideal projective one. This dependence is not derived from a specific microscopic detector model; therefore, the main conclusion of this work is not tied to a unique form of $\beta(\lambda)$ but to the competition between Zeno suppression of dynamics and the growth of the resource losses of the measuring channel.

Full loss-rate functional (action analogue)

The decoherence rate of the qubit state consists of external dephasing, the Zeno term, and the resource losses of the measuring channel, normalized to τ . For the model under consideration, we will take the value $p = 3/2$:

$$G(\tau, \lambda) = \gamma + \gamma_z(\tau, \lambda) + \frac{\beta(\lambda)}{\tau} = \gamma + \frac{\Omega^2 \tau}{4 \lambda} + \frac{\beta_0}{\tau} \frac{1+\lambda}{(1-\lambda)^{3/2}}. \quad (4)$$

For the standard quantum Zeno effect ($\tau \rightarrow 0, \lambda \rightarrow 1$) the functional (4) diverges. As part of further analysis, in the model under consideration, the coherence of a quantum system decreases according to the law $\exp(-Gt)$, with the conservation of coherence time defined as $\sim 1/G$. The stability of the quantum state of a qubit corresponds to the minimum of G . Thus, the functional of the total decoherence rate can be considered as an analogue of the action in the problem of optimizing quantum measurements.

Unlike the standard principle of least action, it is not the path integral of the system that is minimized, but rather the functional of the total decoherence rate. The minimum of the functional $G(\tau, \lambda)$ corresponds to the following:

$$\delta G(\tau, \lambda) = 0, \delta^2 G(\tau, \lambda) > 0,$$

where δ is the variation of the functional.

We will now analyze the minimization of the functional of the decoherence rate of the qubit state.

Minimization of the functional

First variation

For a fixed parameter λ , we calculate the first variation, that is, the partial derivative of the functional $G(\tau, \lambda)$ with respect to τ :

$$\begin{aligned} \frac{\partial G(\tau, \lambda)}{\partial \tau} &= \frac{\Omega^2}{4} \frac{1}{\lambda} - \frac{\beta_0}{\tau^2} \frac{1+\lambda}{(1-\lambda)^{\frac{3}{2}}}, \\ \frac{\Omega^2}{4} \frac{1}{\lambda} - \frac{\beta_0}{\tau^2} \frac{1+\lambda}{(1-\lambda)^{\frac{3}{2}}} &= 0 \Rightarrow \tau_{\text{opt}}^2 = \frac{4\beta_0\lambda}{\Omega^2} \frac{1+\lambda}{(1-\lambda)^{\frac{3}{2}}}. \end{aligned} \quad (5)$$

Equation (5) yields the expression for τ_{opt} . Similarly, we find the partial derivative of the functional $G(\tau, \lambda)$ with respect to λ :

$$\frac{\partial G(\tau, \lambda)}{\partial \lambda} = -\frac{\Omega^2}{4} \frac{\tau}{\lambda^2} + \frac{\beta_0}{\tau} \frac{5+\lambda}{2(1-\lambda)^{\frac{5}{2}}} = 0. \quad (6)$$

Multiplying both parts of equation (6) by 2τ , we obtain:

$$\frac{\Omega^2}{2} \frac{\tau^2}{\lambda^2} = \beta_0 \frac{5+\lambda}{(1-\lambda)^{\frac{5}{2}}}. \quad (7)$$

We substitute τ_{opt}^2 into expression (7):

$$\frac{2\beta_0}{\lambda} \frac{1+\lambda}{(1-\lambda)^{\frac{3}{2}}} = \beta_0 \frac{5+\lambda}{(1-\lambda)^{\frac{5}{2}}}. \quad (8)$$

After mathematical transformations of expression (8), we obtain the following:

$$\frac{5+\lambda}{(1-\lambda)^{\frac{5}{2}}} = \frac{2(1+\lambda)}{\lambda(1-\lambda)^{\frac{3}{2}}}. \quad (9)$$

Expression (9) is transformed into a quadratic equation:

$$3\lambda^2 + 5\lambda - 2 = 0. \quad (10)$$

The numerical solution of equation (10) leads to a positive root $\lambda_{\text{opt}} = \frac{1}{3}$ at τ_{opt} :

$$\tau_{\text{opt}} = \frac{2}{\Omega} \sqrt{\beta_0 \lambda \frac{1+\lambda}{(1-\lambda)^{\frac{3}{2}}}}.$$

The obtained values were verified using the built-in Given–Find tool for solving nonlinear equations in the Mathcad environment.

The accuracy of calculations in Given–Find is determined by the variables α and μ . The variable α determines the absolute error of the calculation. For the equation $f(x) = g(x)$ the solution is considered found if the following condition is met:

$$|f(x) - g(x)| \leq \alpha.$$

The variable μ (convergence tolerance) controls the criterion of the iterative process:

$$\frac{|x_{k+1} - x_k|}{x_{k+1}} \leq \mu,$$

where k is the step between iterations, and x_k is the root of the equation at the k^{th} step.

When solving expression (4), the following values were used: $\gamma = 1$, $\Omega = 1$, $\beta_0 = 0.5$, $\alpha = \mu = 10^{-3}$. For the initial approximation, the values were as follows: $\tau = 1$, $\lambda = 1$. The values of these parameters were selected for a qualitative analysis of the functional under consideration. To improve reproducibility, several initial approximations in the domain $\tau > 0$ and $0 < \lambda < 1$ were tested. The iteration converged to the same positive stationary point, whereas nonphysical roots with λ outside the admissible interval or $\tau \leq 0$ were discarded.

Among the obtained solutions, the positive roots have physical meaning: $\tau_{\text{opt}} = 1.278$ and $\lambda_{\text{opt}} = \frac{1}{3}$. The minimum of the functional for τ_{opt} and λ_{opt} corresponds to the value $G(\tau_{\text{opt}}, \lambda_{\text{opt}}) \cong 2.917$. The solution is stable under variations of the initial approximations, which indicates the existence of a single physically significant minimum in the region $0 < \lambda < 1$. Fig. 1 shows the dependence of the functional $G(\tau, \lambda)$ on the parameters τ and λ .

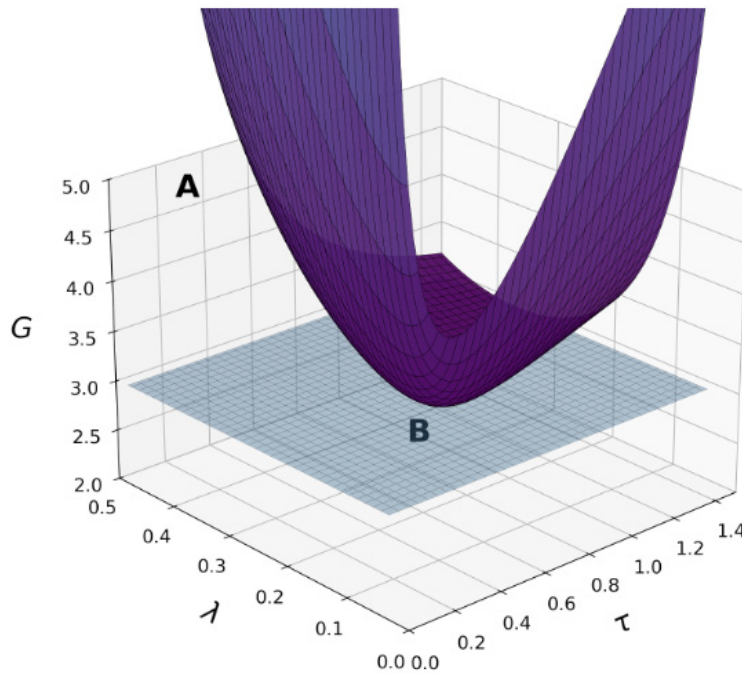


Fig. 1. A — dependence of the functional G on the measurement time interval τ and the measurement imperfection λ . B — level plane corresponding to the minimum of the functional

Thus, the analysis of the obtained results shows that the values τ_{opt} and λ_{opt} actually correspond to the minimum of the functional G .

Check of the second variation

According to the stability principle, the second variation of the functional must be positive. The second derivatives have the form:

$$\begin{aligned}\frac{\partial^2 G(\tau, \lambda)}{\partial \tau^2} &= \frac{\beta_0}{\tau^3} \frac{1+\lambda}{(1-\lambda)^{3/2}} > 0 \\ \frac{\partial^2 G(\tau, \lambda)}{\partial \lambda^2} &= \frac{\Omega^2}{2} \frac{\tau}{\lambda^3} + \frac{\beta_0}{\tau} \frac{3(9+\lambda)}{4(1-\lambda)^2} > 0.\end{aligned}\quad (11)$$

The result (11) indicates that the presence of a minimum of the functional $G(\tau, \lambda)$ acts as a criterion for the stability of the quantum state of the qubit.

Comparison with near-perfect measurements

Functional (4) for the value τ_{opt} corresponds to the following expression:

$$G_{\min}(\lambda)|_{\tau_{\text{opt}}} = \gamma + \frac{\Omega^2}{4} \frac{1}{\lambda} \frac{2\sqrt{\lambda\beta(\lambda)}}{\Omega} + \beta(\lambda) \frac{1+\lambda}{2\sqrt{\lambda\beta(\lambda)}}.$$

After the transformation $G_{\min}(\lambda)|_{\tau_{\text{opt}}}$ we arrive at the following:

$$G_{\min}(\lambda)|_{\tau_{\text{opt}}} \cong \gamma + \Omega \sqrt{\frac{\beta(\lambda)}{\lambda}}. \quad (12)$$

Expression (12) is interpreted as the minimum decoherence rate achieved at the optimal measurement frequency.

In the limit $\lambda_{\text{im}} \rightarrow 1$ (ideal measurements), expression (12) takes the following form:

$$G_{\min}(\lambda_{\text{im}}) = \gamma + \Omega \sqrt{\frac{\beta_0}{\lambda} \frac{1+\lambda}{(1-\lambda)^{3/2}}} \approx \gamma + 45 \cdot \Omega \sqrt{\beta_0}.$$

When $\lambda_{\text{opt}} = 1/3$, expression (12) is simplified to the following form:

$$G_{\min}(\lambda_{\text{opt}}) = \gamma + \Omega \sqrt{\frac{\beta_0}{\lambda} \frac{1+\lambda}{(1-\lambda)^{3/2}}} \approx \gamma + 3 \cdot \Omega \sqrt{\beta_0}.$$

Using relation (3), we write the following:

$$\frac{G_{\min}(\lambda_{\text{im}})}{G_{\min}(\lambda_{\text{opt}})} \approx \frac{46 \cdot \Omega \sqrt{\beta_0}}{4 \cdot \Omega \sqrt{\beta_0}} \approx 11.5. \quad (13)$$

Expression (13) indicates that, for the selected set of parameters, the qubit coherence time increases by a factor of 11.5 compared with the ideal-measurement limit. This estimate is not a universal constant: it is determined by the chosen values of γ , Ω , and β_0 , and must be recalculated when these physical scales change.

Discussion of the results

Within the model in question, the value $\lambda_{\text{opt}} = 1/3$ indicates that the minimum of the qubit decoherence-rate functional is achieved at a finite value of the measurement imperfection parameter. Thus, in this model, imperfect measurements can be more favorable than the ideal-measurement limit for stabilizing the qubit state. The conclusion is model-dependent and should not be interpreted as a universal statement for all quantum-measurement schemes. The sensitivity to the exponent p in the loss term was also analyzed. If p is left arbitrary in $\beta(\lambda)$, minimization over λ leads to the condition $p\lambda^2 + (1+p)\lambda - 1 = 0$. Therefore, for $p > 0$ the optimal value of λ remains finite and lies in the interval $0 < \lambda < 1$. Changing p shifts the position of the minimum and the numerical gain in coherence time, but does not remove the finite optimum as long as the measurement-channel losses grow when the ideal-projective-measurement limit is approached. The value $p = 3/2$ is therefore used as a representative model case rather than as a universal exponent.

In contrast to the works (Belan, Parfenyev 2020; Hacohen-Gourgy, Martin 2020; Jacobs, Steck 2006), which investigate optimal modes of Zeno quantum dynamics and methods of continuous quantum monitoring, the present work optimizes the stability of a quantum system simultaneously with respect to the measurement time interval and the measurement imperfection parameter. Thus, the measurement imperfection parameter can be included among the control parameters of quantum control in real experimental setups, where ideal measurements are limited by the finite characteristics of measuring devices. Promising directions for further research include:

- Derivation of loss functional from microscopic models of detectors;
- Numerical simulation of open quantum systems;
- Analysis of different noise statistics and detector-feedback mechanisms;
- Analysis of multi-qubit systems.

The main result of the work is the demonstration of competition between Zeno suppression of dynamics and the resource costs of the measuring channel. The conclusion is expected to remain valid in regimes where environmental noise can be represented as an additional positive decoherence channel and where measurement resources grow strongly near the ideal projective-measurement limit.

Strongly non-Markovian noise, correlated detector back-action, feedback from the measuring device, or multi-qubit interactions may quantitatively change the position of the minimum and, in some architectures, modify the stability criterion itself. In such cases, the optimum must be recalculated for the specific experimental platform.

Thus, the obtained result should be considered as a model demonstration that finite measurement imperfection can increase the stability of the qubit state under realistic resource constraints.

Conclusions

In this paper, we introduced a functional for the decoherence rate of a qubit state, optimizing it with respect to the measurement time interval and the measurement imperfection parameter. As part of the studied model, the stability principle is used as an optimization criterion.

The obtained results are consistent with what is observed in quantum measurement theory: achieving the most ideal measurements is accompanied by increased resource consumption. In the model under consideration, this leads to a tradeoff between the effectiveness of Zeno's dynamic suppression and the losses in the measurement channel.

Calculations show that, for the selected set of parameters, the optimal measurement mode results in an 11.5-fold increase in coherence time compared with the ideal-measurement limit. This value is illustrative and depends on the physical scales of the model. The results indicate that finite measurement imperfection can be used as a control parameter for increasing the stability of the qubit state.

Conflict of Interest

The authors declare that there is no conflict of interest, either existing or potential.

Author Contributions

Timur F. Kamalov developed a methodology for analyzing the stability principle of a qubit and provided guidance for writing the manuscript; Andrey A. Borisov and Anton A. Borisov formulated the physical problem and conducted numerical calculations.

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