

UDC 517.938

DOI: 10.33910/2687-153X-2020-1-4-158-164

A model of a quantum waveguide multiplexer

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For citation: Vorobiev, A. M., Yurova, T. S., Popov, I. Yu. (2020) A model of a quantum waveguide multiplexer. *Physics of Complex Systems*, 1 (4), 158–164. DOI: 10.33910/2687-153X-2020-1-4-158-164

Received 10 October 2020; reviewed 5 November 2020; accepted 5 November 2020.

Funding: This work was partially financially supported by the Government of the Russian Federation (grant 08-08), by grant 16-11-10330 of Russian Science Foundation.

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Abstract. The paper explores the system of quantum waveguides and resonators. It suggests a solvable model of zero-width coupling windows based on the operator extensions theory in the Pontryagin space with indefinite metrics. The model self-adjoint operator is constructed explicitly, and it is similar (in some sense) to that of a physical system. We obtain an expression for the transmission coefficient for electrons and investigate its dependence on the electron energy which has a resonant character. It allows to control the electron transmission to different waveguides. The paper proposes a model of a three-channel quantum mesoscopic multiplexer.

Keywords: quantum waveguide, transmission, resonance, operator extensions theory, mesoscopic system.

Introduction

Development of nanotechnology and miniaturization of electronic devices causes the necessity to take into account quantum properties of electron. It is a challenge, yet, it opens a possibility of designing novel devices based on quantum principles. One of such promising developments is a quantum computer. Currently, there are a few quantum computers in operation, however, they are very complicated and are not effective enough. Existing quantum computers can operate with a small number of quantum bits. The scaling of such devices is a great challenge. A possible solution to this challenge lies in the background improvements, e.g. in the use of triadic logic devices. The paper proposes a quantum multiplexer for the triadic logic device and develops a model to describe its processes. It deals with a system of coupled quantum waveguides (quantum wires) and quantum resonators (quantum dots). To describe the electron transmission through the system, we suggest using a model of zero-width coupling windows which has some advantages. Namely, it is solvable, i. e. it yields analytical results. On the other hand, its properties are similar to those of the corresponding physical system. The model has proven to be effective in many cases, see (Pavlov et al. 2001; Popov, Popova 1993a).

The paper focuses on two-dimensional quantum mesoscopic systems, i. e. the physical systems where the electron phase coherence is preserved at a scale much larger than the atomic dimensions. For such systems, quantum properties play an important role (Landauer 1957; Sols et al. 1989; Takagaki, Ploog 1994). The studies of low-dimensional systems usually rely on the Landauer-Buttiker theory (Beenakker, van Houten 1991; Buttiker 1993; Landauer 1970) with its direct relation between the conductivity and the transmission coefficient for ballistic channels.

We consider waveguides connected with resonators through small apertures. It is very difficult to give a full description of such system (Exner, Kovarik 2015; Nazarov et al. 2019). For this reason, we propose an explicitly solvable model which preserves the main features of the corresponding real system. The model is analogous to the zero-range potential approach in quantum mechanics (Albeverio et al. 2005; Pavlov 1987). It is based on the theory of self-adjoint extensions of symmetric operators. The model of resonators coupled through small openings was suggested in (Melikhova, Popov 2017; Popov 1992; 1997; 2013). Resonance effects in the waveguide-resonator-waveguide system in the framework of the model were the focus of (Popov, Popova 1993a; 1993b).

The paper is structured as follows: the first section provides a brief overview of a general mathematical scheme; section two focuses on the implementation of the model for the system of a branching quantum waveguide with inserted quantum resonators; finally, we discuss a possibility to apply the model to the development of a nanoelectronic device.

General mathematical scheme

The conventional operator extension theory approach to the description of resonators coupled through small windows may be found in (Vorobiev et al. 2019). Let us consider two domains Ω_1, Ω_2 with smooth boundaries having a common point r_0 . The initial self-adjoint operator for the model construction is the orthogonal sum of the Laplace operators in the domains. The next steps depend on the boundary conditions. In the case of the Neumann boundary conditions, the restriction of the operator on the set of functions vanishing at point r_0 gives a symmetric non-self-adjoint operator with deficiency indices (2,2). Correspondingly, a self-adjoint extension of the operator may be constructed. It is the operator which gives us the model in question. In the case of the Dirichlet boundary conditions, the restricted operator is essentially self-adjoint, i. e. it has deficiency indices (0,0). To construct the model for this boundary condition, it is necessary to extend the space L_2 by adding elements with greater singularities at the point r_0 . As a result, we have to deal with the Pontryagin space.

Let us briefly describe the model for the Dirichlet case (Popov 1992) using an indefinite scalar product. Let $\Delta^{1,2}$ be the Laplace operators with the Dirichlet boundary conditions in the domains $\Omega^{1,2}$ with smooth boundaries having a common point r_0 . There are two ways of including functions with higher singularities into the space: either considering weighted space or deal with an indefinite scalar product. We follow the latter. Let us consider the following function:

$$h_{-1}^{1,2} = \frac{\partial}{\partial n_y} G^{1,2}(r, r', k_0) \Big|_{r'=r_0},$$

which does not belong to L_2 . Here, $G^{1,2}$ is the Green function for the Dirichlet problem in $\Omega^{1,2}$. Let us define the following set of functions:

$$A^{1,2} = \left\{ f(r) : f \in L_2(\Omega^{1,2}), \int_{\Omega^{1,2}} |f(r)| |r - r_0|^{-2} dr < \infty \right\}.$$

Functions from this set have roots of at least first multiplicity at r_0 . It is necessary to ensure the integral convergence. Let us consider a pair of functions:

$$h_{-1}^{1,2}, \quad h_1^{1,2} = (-\Delta^{1,2} - \lambda_0)^{-1} h_{-1}^{1,2}.$$

Here k_0 is some imaginary number ($k_0^2 = \lambda_0$ is a regular point of the Laplace operator in $\Omega^{1,2}$). Now, we introduce another set of functions having singularities $\tilde{A}^{1,2}$:

$$\tilde{A}^{1,2} = \left\{ \tilde{f}^{1,2} : \tilde{f}^{1,2} = f^{1,2} + C_1^{1,2} h_1^{1,2} + C_{-1}^{1,2} h_{-1}^{1,2} \right\}.$$

Here, the first term in the right-hand side is smooth: $f^{1,2} \in \tilde{A}^{1,2}$. It is necessary to define a scalar product in $\tilde{A}^{1,2}$. It can be made in the following way:

$$\begin{aligned} (\tilde{f}, \tilde{g})_{\tilde{A}^{1,2}} &= (f, g)_{L_2} + f_{\Omega^{1,2}} \int_{\Omega^{1,2}} f(r) \overline{(C_1^g h_1^{1,2} + C_{-1}^g h_{-1}^{1,2})} + f_{\Omega^{1,2}} g(r) (C_1^f h_1^{1,2} + C_{-1}^f h_{-1}^{1,2}) dr \\ &+ C_1^f \overline{C_1^g} (h_1^{1,2}, h_1^{1,2})_{L_2} + (C_1^f C_{-1}^g + C_{-1}^f \overline{C_1^g}) (h_{-1}^{1,2}, h_1^{1,2})_{L_2}. \end{aligned}$$

The set $\tilde{A}^{1,2}$ is not yet a full space. It is embedded into the Pontryagin space Π_1 by conventional way (Derkach et al. 2003; Popov 1992; Shondin 1988; van Diejen, Tip 1991).

Now, it is necessary to define the Laplace operator in this extended space. Let the domain of the operator $\tilde{\Delta}^1$ have the form:

$$D(\tilde{\Delta}^1) = \left\{ \check{f} : \check{f} \in \tilde{A}^1, f \in W_2^{2,loc}(\Omega^1), f = f_1 + Ch_1^1 \right\}.$$

Here, $f_1, (-\tilde{\Delta}^1 - \lambda_0)f_0 \in A^1$. On the set A^1 the operator $\tilde{\Delta}^1$ acts as the Laplace operator and on the chain h_p^1 the operator $(-\tilde{\Delta}^1 - \lambda_0)$ is a shift operator: $(-\tilde{\Delta}^1 - \lambda_0)h_1^1 = h_{-1}^1$. One can prove that the operator $-\tilde{\Delta}^1$ is self-adjoint.

The next step of the model construction is the so-called “restriction-extension” procedure. Let us restrict the operator $-\tilde{\Delta}^1$ onto the following set:

$$D(\Delta_0^1) = \left\{ f : f \in D(\tilde{\Delta}^1), ((-\tilde{\Delta}^1 - \lambda_0)f, h_{-1}^1) = 0 \right\}.$$

The obtained operator Δ_0^1 is symmetric and has deficiency indices (1,1) in the Pontryagin space. The construction procedure for the second domain for Ω^2 is the same. Correspondingly, the orthogonal sum $\Delta_0 = \Delta_0^1 \oplus \Delta_0^2$ is a symmetric operator with deficiency indices (2,2). Its self-adjoint extensions are restrictions of the adjoint operator. It is easy to describe the domain of the adjoint operator:

$$D(\tilde{\Delta}_0^*) = \left\{ \check{f} : \check{f} \in \tilde{A}^1, f = f_1 + Ch_1^1 \right\}.$$

Here, $f_1, (-\tilde{\Delta}_0^* - \lambda_0)f_1 \in A^1$. To construct a self-adjoint extension one should find a linear set of elements from $D(\tilde{\Delta}_0^*)$ satisfying the self-adjointness condition:

$$J(f, g) = \left((-\tilde{\Delta}_0^* - \lambda_0)\check{f}, \check{g} \right) - \left(\check{f}, (-\tilde{\Delta}_0^* - \lambda_0)\check{g} \right).$$

One can see that elements of $D(\Delta_0)$ are such elements from $D(\tilde{\Delta})$ that satisfy the condition $C_1^1 = C_1^2 = 0$. Taking into account the known asymptotics of the Green function near the boundary point, we obtain the following:

$$J(f, g) = C_1^{f,1} \overline{C_{-1}^{g,1}} - C_{-1}^{f,1} \overline{C_1^{g,1}} + C_1^{f,2} \overline{C_{-1}^{g,2}} - C_{-1}^{f,2} \overline{C_1^{g,2}}.$$

Geometrical treatment of Lagrange planes leads to the following statement:

Theorem 1. *The set of self-adjoint extensions of the operator $-\Delta_0$ consists of the sets of operators $\{-\Delta_A\}, \{-\Delta_B\}$, the domains of which consist of all elements from $D(\Delta_0^*)$ satisfying the conditions*

$$\begin{pmatrix} C_1^1 \\ C_1^2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} C_{-1}^1 \\ C_{-1}^2 \end{pmatrix}$$

or

$$\begin{pmatrix} C_{-1}^1 \\ -C_{-1}^2 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} C_1^1 \\ C_{-1}^2 \end{pmatrix},$$

inhere A, B are Hermitian matrices.

Remark. We have no goal to describe all possible extensions. We deal with one extension only corresponding to the following matrix (Popov 1992).

$$b_{11} = b_{22} = 0, b_{12} = b_{21} = -1 \tag{1}$$

The system of coupled waveguides and resonators

Let us consider the problem of electron scattering in the system of coupled quantum waveguides and resonators (Fig. 1). The construction of the model operator is the same as in the case of the two coupled domains described above.

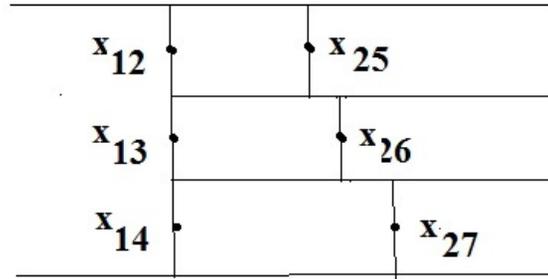


Fig. 1. The geometry of the system. Coupling points are marked

We choose “the most natural” extension (see remark at the end of the previous section). In this case the solution of the scattering problem for the model operator has the form:

$$\psi(r, k) = \left\{ \begin{array}{l} \psi_0(r, k) + \alpha_{12} \frac{\partial G_1}{\partial x}(r, r_{12}, k) + \alpha_{14} \frac{\partial G_1}{\partial x}(r, r_{14}, k), \\ -\alpha_{12} \frac{\partial G_2}{\partial x}(r, r_{12}, k) - \alpha_{25} \frac{\partial G_2}{\partial x}(r, r_{25}, k), \\ -\alpha_{13} \frac{\partial G_3}{\partial x}(r, r_{13}, k) - \alpha_{36} \frac{\partial G_3}{\partial x}(r, r_{36}, k), \\ -\alpha_{14} \frac{\partial G_4}{\partial x}(r, r_{14}, k) - \alpha_{47} \frac{\partial G_4}{\partial x}(r, r_{47}, k), \\ \alpha_{25} \frac{\partial G_5}{\partial x}(r, r_{25}, k), \\ \alpha_{36} \frac{\partial G_6}{\partial x}(r, r_{36}, k), \\ \alpha_{47} \frac{\partial G_4}{\partial x}(r, r_{47}, k), \end{array} \right. \quad (2)$$

Here $G(r, r, k)$ is the Green function for the Dirichlet problem in $\Omega, r = (x, y)$, $\psi_0(r, k)$ is the solution of the scattering problem for a semi-infinite waveguide Ω_1 without point-like coupling windows (the sum of incoming and reflected waves), α_{mn} are coefficients to be determined. Taking into account, Theorem 1 and (1) one obtains the following linear system for α_{mn} :

$$\begin{aligned} \alpha_{12}g_1(r_{12}, k) + \alpha_{13} \frac{\partial G_1}{\partial x}(r_{12}, r_{13}, k) + \alpha_{14} \frac{\partial G_1}{\partial x}(r_{12}, r_{14}, k) + \frac{\partial \psi_0}{\partial x}(r_{12}, k) \\ = \alpha_{12}g_1(r_{12}, k) + \alpha_{25} \frac{\partial G_2}{\partial x}(r_{12}, r_{25}, k), \\ \alpha_{25}g_2(r_{25}, k) + \alpha_{12} \frac{\partial G_2}{\partial x}(r_{25}, r_{12}, k) = \alpha_{25}g_5(r_{25}, k), \\ \alpha_{13}g_1(r_{13}, k) + \alpha_{12} \frac{\partial G_1}{\partial x}(r_{13}, r_{12}, k) + \alpha_{14} \frac{\partial G_1}{\partial x}(r_{13}, r_{14}, k) + \frac{\partial \psi_0}{\partial x}(r_{13}, k) \\ = \alpha_{13}g_3(r_{13}, k) + \alpha_{36} \frac{\partial G_3}{\partial x}(r_{36}, r_{13}, k), \end{aligned} \quad (3)$$

$$\begin{aligned} \alpha_{36}g_3(r_{36},k) + \alpha_{13} \frac{\partial G_3}{\partial x}(r_{36},r_{13},k) &= \alpha_{36}g_6(r_{36},k), \\ \alpha_{14}g_1(r_{14},k) + \alpha_{12} \frac{\partial G_1}{\partial x}(r_{14},r_{12},k) + \alpha_{13} \frac{\partial G_1}{\partial x}(r_{14},r_{13},k) + \frac{\partial \psi_0}{\partial x}(r_{14},k) \\ &= \alpha_{14}g_4(r_{14},k) + \alpha_{47} \frac{\partial G_4}{\partial x}(r_{14},r_{47},k), \\ \alpha_{47}g_4(r_{47},k) + \alpha_{14} \frac{\partial G_4}{\partial x}(r_{14},r_{47},k) &= \alpha_{47}g_7(r_{47},k), \end{aligned}$$

here

$$g_i(r_{jl},k) = \left(\frac{\partial G_i}{\partial x}(r,r_{jl},k) - \frac{\partial G_i}{\partial x}(r,r_{jl},k_0) \right) \Big|_{r \rightarrow r_{jl}},$$

k is a model parameter. To choose it, one can compare the solutions of the model and realistic problems. This analysis showed (Popov 1992; 2013) that small width corresponds to large k_0 . Moreover, it is possible to choose the parameter in such a way that the model solution gives one the main asymptotic term (in the width of the window) of the corresponding realistic problem. We assume that all coupling windows are identical, in opposite case, it is necessary to introduce several parameters k_{0j} (one for each window). The expression for $\frac{\partial G_i}{\partial x}(r,r',k)$ for a semi-infinite waveguide is well known:

$$\begin{aligned} \frac{\partial G_i}{\partial x}(r,r',k) = \frac{\partial}{\partial x'} \sum_{n=1}^{\infty} \frac{2 \sin \frac{\pi n y}{d_j} \sin \frac{\pi n y'}{d_j}}{d_j \sqrt{\frac{\pi^2 n^2}{d_j^2} - k^2}} &\left(\exp \left(i \sqrt{\frac{\pi^2 n^2}{d_j^2} - k^2} |x - x'| \right) \right. \\ &\left. - \exp \left(i \sqrt{\frac{\pi^2 n^2}{d_j^2} - k^2} |x - x^*| \right) \right), \end{aligned}$$

where x^* is a mirror image of the point x with respect to the end of the waveguide. Expression for $g_2(r_{12},k)$ (and, correspondingly, for $g_4(r_{14},k)$, $g_3(r_{13},k)$) is

$$g_2(r_{12},k) = (k^2 - k_0^2) \sum_{n,m=1}^{\infty} \frac{4\pi^2 n^2 \sin^2 \frac{\pi m}{2}}{d_5 l_2^3 \left(\frac{\pi^2 n^2}{l_2^2} + \frac{\pi^2 m^2}{d_5^2} - k^2 \right) \left(\frac{\pi^2 n^2}{l_2^2} + \frac{\pi^2 m^2}{d_5^2} - k_0^2 \right)}.$$

One can solve system (3) and obtain coefficients α_{25} , α_{36} , α_{47} , which give us the transmission coefficients for channels Ω_5 , Ω_6 , Ω_7 correspondingly.

Discussion

Let us consider the particular case for which the calculations were made. Let $d_5 = d_6 = d_7 = d/3$, and l_2, l_3, l_4 differs slightly. Let us compute transmission coefficients, i. e. the squares of modulus of coefficients α_{25} , α_{36} , α_{47} , as functions of the electron energy (k^2) for this particular case to show the possibility of using the suggested construction. The dependences have a resonant character. There are resonant peaks when k^2 is close to an eigenvalue of the corresponding resonator (see Fig. 2). The calculations were made for the first two resonant peaks for $d_5 = d_6 = d_7 = 3$, $l_2 = 1$, $l_3 = 1.1$, $l_4 = 1.2$ (in dimensionless units). This effect can be used for construction of a mesoscopic quantum device.

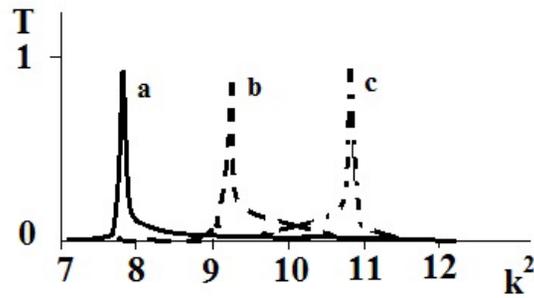


Fig. 2. Transmission coefficients to waveguides 7 (a), 6 (b), 5 (c) via the electron energy k^2 (dimensionless units)

Namely, if electron energy is close an eigenvalue of the resonator Ω we have a resonant transmission to the channel Ω and almost zero transmissions to other channels.² Varying electron energy we can obtain resonant⁵ transmission to the channel Ω (or Ω) with zero transmissions to other channels. We choose resonators with sizes which change slightly. Hence, small variation of k^2 leads to switching of a signal from one channel to another. Here the control parameter is electron energy.

Another control parameter can be used as well. For example, one can control resonant condition by changing parameters of resonators, or widths of coupling apertures. This can be made by varying the corresponding bias voltage.

In the suggested construction we use resonant properties of coupled resonators, but it is possible to use resonant properties of coupling apertures itself. The existence of such properties was shown for a system of waveguides coupled through small windows in (Popov, Popova 1993a; 1993b) (for the model) and in (Vorobiev et al. 2019) (for a realistic problem). A possible construction based on this effect will be developed and described in the nearest future.

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