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Non-linearity of Vaidya spacetime and forces in the central naked singularity

V. D. Vertogradov[1,2]

[1] Herzen State Pedagogical University of Russia, 48 Moika Emb., Saint Petersburg 191186, Russia
[2] The Special Astrophysical Observatory of the Russian Academy of Sciences, 65 Pulkovskoe Highway, Saint Petersburg 196140, Russia

Authors
Vitalii D. Vertogradov, ORCID: 0000-0002-5096-7696, e-mail: vdvertogradov@gmail.com

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Abstract. The paper focuses on non-linear Vaidya spacetime, i. e., cases when the mass function has a non-linear form $M(v) \equiv \lambda v^n$, $\lambda > 0$, $n > 1$. We prove that the central naked singularity might form for values $n > 1$, however, under such conditions it will be gravitationally weak. The paper also disucsses forces in the naked singularity and proves that they might be finite only in the case of the gravitationally weak naked singularity. It also focuses on the strength of the singularity. We prove that the strong naked singularity might form only in the linear case. The singularity might also be gravitationally strong in the case of black hole formation.

Keywords: Vaidya spacetime, naked singularity, strength of singularity, inertial forces, geodesics, gravitational collapse

Introduction

The Vaidya spacetime is one of the earliest counterexamples of cosmic censorship conjecture [CCC] violation (Vaidya et al. 1985). Vaidya spacetime is so-called radiating Schwarzschild metric with non-zero right hand-side of the Einstein equation. The energy-momentum tensor in Vaidya spacetime represents the null dust and has the following form:

$$T_{ik} = \rho \delta_i^0 \delta_k^0 \,, \tag{1}$$

where ρ is the energy density of this null dust.

The gravitational collapse of usual Vaidya spacetime (Dwivedi, Joshi 1989) might lead to the formaiton of the naked singularity. It means that there exist a family of non-spacelike future-directed geodesics which terminate in the central singularity in the past. Moreover, the time of the singularity formation must be shorter than the time of the apparent horizon formation. The geodesic motion represents the free movement. It means there is no any non-gravitational force acting on the particle. However, there are so-called inertial forces and the particle moves in the medium because this spacetime is filled with the null dust. We can calculate these forces and, as we see below, in the case of linear Vaidya spacetime (the mass function is the linear one) these forces are infinite. We will show that in the case of non-linear Vaidya spacetime these forces might be finite.

We will provide evidence that the naked singularity which is formed in the case of non-linear Vaidya spacetime is gravitationally weak. Let us consider Tipler's definition given in the paper (Nolan 1999): a singularity is termed to be gravitationally strong or simply strong if it destroys by stretching or crushing any object which falls into it. If it does not destroy any object this way, then the singularity is termed to be gravitationally weak.

The paper investigates the inertial forces in the case of the naked singularity formation and determines the conditions under which these forces might be finite. The paper also focuses on the strength of the singularity.

This paper is organised as follows: in Sec. II we consider the possibility of the naked singularity formation in the case $M(v) = \lambda v^n$, in Sec. III we consider the inertial forces in Vaidya spacetime. In Sec. IV we investigate the strength of the naked singularity in the case of non-linear Vaidya metric. Sec. V is the conclusion.

Throughout the paper the system of units $c = G = 1$will be used.

Naked singularity formation

The usuall Vaidya spacetime has the following form:

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right)dv^2 + 2\varepsilon dvdr + r^2 d\Omega^2, \tag{1}$$

where $\varepsilon = \pm 1$is ingoing (outgoing) radiation, $M(v)$ is the mass function, and $v_{\pm 1}$ is advanced (retarded) Edington's time. $d\Omega$ is the metric on the unit two-sphere:

$$d\Omega_2 \equiv d\theta_2 + sin_2\, \theta d\phi_2. \tag{2}$$

Our case is related to the gravitational collapse, hence, we will use $\varepsilon = +1$ throughout the paper.

The case $M(v) = \lambda v^n$

Let us consider the case when the mass function has the form:

$$M(v) = \lambda v^n, \tag{3}$$

where λ is a positive real constant.

The equation of the apparent horizon is given by (Poisson 2004):

$$g_{00} = \frac{2\lambda v^n}{r} - 1 = 0\ . \tag{4}$$

We can see that if $v = 0$, then we can not satisfy the apparent horizon equation (we need positive values of r).

At the time $v = 0$ and at the point $r = 0$ we have the singularity formation but for it to be naked there must be non-spacelike future-directed geodesics which terminate at the central singularity in the past. Let us consider the existence of radial null geodesics. The geodesic equation in this case is given by:

$$\frac{dv}{dr} = \frac{2r}{r - 2\lambda v^n}\ . \tag{5}$$

The geodesic can originate from the central singularity if $\lim_{v \to 0, r \to 0} \frac{dv}{dr} = X_0$, where X_0 is a finite positive number.

We should consider 3 cases:

- $0 < N < 1$,
- $n = 1$,
- $n > 1$.

The case of linear mass function $n = 1$ has been considered in (Dwivedi, Joshi 1989). In this case we have the naked singularity formation if $\lambda < \frac{1}{8}$. Also, this singularity is gravitationally strong (Nolan 1999; Tipler 1977).

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Let us consider the case $n < 1$. Let us denote $\lim\limits_{v\to 0, r\to 0} \frac{dv}{dr} = X_0$, then

$$X_0 = \frac{2}{1-2\lambda n X_0 v^{n-1}}, \qquad 2\lambda n v^{n-1} X_0^2 - X_0 + 2 = 0 . \tag{6}$$

$v \to 0$ and $n < 1$, hence, from the last equation we can see X_0 can not be positive real constant and $X_0 \to \infty$. We can conclude that, in this case, there are no radial null geodesics which terminate at the central singularity in the past.

Now, let us consider the case when $n > 1$. Thus, we have:

$$X_0 = \frac{2}{1-2\lambda n X_0 v^{n-1}}, \qquad 2\lambda n v^{n-1} X_0^2 - X_0 + 2 = 0 . \tag{7}$$

Here, we have three possible options:

• $X_0 = 0$. In this case we do not have any uncertainty and the geodesic equation (7) gives impossible equality. So, this option is impossible.

• $X_0 \to \infty$. In this case the condition $\lim\limits_{r\to 0\, v\to 0} v^{n-1} X_0 = \frac{1}{2\lambda n}$ must be held. However, the previous equation (7) gives impossible equality, which is why this option is also unacceptable.

• $X_0 = \mu$, where μ is real positive constant. In this case we have an equality $X_0 = 2$ and it is the only suitable option.

Note that here we have a radial null geodesic which terminates at the central singularity in the past regardless of value . Further we will show that in the case $n > 1$ this singularity is gravitationally weak.

The inertial forces

To investigate the inertial forces one should consider the second order geodesic equations. In Vaidya spacetime they are given by:

$$\frac{d^2t}{d\tau^2} = -\frac{M(v)}{r^2}\left(\frac{dt}{d\tau}\right)^2 + r\left(\frac{d\theta}{d\tau}\right)^2 + r\sin^2\theta\left(\frac{d\varphi}{d\tau}\right)^2 . \tag{8}$$

$$\frac{d^2r}{d\tau} = -\frac{\left(1-\frac{2M(v)}{r}\right)M(v)+\dot{M}(v)r}{r^2}\left(\frac{dt}{d\tau}\right)^2 + 2\frac{M(v)}{r^2}\frac{dt}{d\tau}\frac{dr}{d\tau} + \left(r - \ M(v)\right)\left(\frac{d\theta}{d\tau}\right)^2 + \left(r - \ M(v)\right)\sin^2\theta\left(\frac{d\varphi}{d\tau}\right)^2 . \tag{9}$$

$$\frac{d^2\theta}{d\tau^2} = -\frac{2}{r}\frac{dr}{d\tau}\frac{d\theta}{d\tau} + \sin\theta\cos\theta\left(\frac{d\varphi}{d\tau}\right)^2 . \tag{10}$$

$$\frac{d^2\varphi}{d\tau^2} = -\frac{2}{r}\frac{dr}{d\tau}\frac{d\varphi}{d\tau} - 2\cot\theta\frac{d\theta}{d\tau}\frac{d\varphi}{d\tau} . \tag{11}$$

One should note that only the following combination of four-velocity $u^0 u^0$ and $u^0 u^\alpha$, $\alpha=1, 2, 3$ gives us the inertial forces. The combination $u^\alpha u^\beta$ is the part of three covariant derivative and they are not forces at all (Landau, Lifshitz 1980).

Here we have 2 different forces $\Gamma^0_{00} = -\Gamma^1_{01} = \frac{M(v)}{r^2}$ and $\Gamma^1_{00} = \frac{\left(1-\frac{2M(v)}{r}\right)M(v)+\dot{M}(v)r}{r^2}$. We are interested in the case of the naked singularity formation. As we found out above, under the mass function $M(v) = \lambda v^n$, the gravitational collapse will result in the naked singularity when $n \geq 1$. Using the well-known formula (Landau, Lifshitz 1980):

$$u^{\alpha} = \frac{1}{\sqrt{1-\beta^2}} \frac{dx^{\alpha}}{dt} ,$$
$$u^{0} = \frac{1}{\sqrt{1-\frac{2\lambda v^n}{r}\left(1-\beta^2\right)}} - \frac{r}{\left(r-2\lambda v^n\right)\left(1-\beta^2\right)} \frac{dx^{\alpha}}{dt} ; \tag{12}$$

we can easily obtain the radial force expression:

$$F^{r}_{cent} = -\frac{\left(1-\frac{2\lambda v^n}{r}\right)\lambda v^n + \lambda n v^{n-1} r}{r^2\left(1-\frac{2\lambda v^n}{r}\right)\left(1-\beta^2\right)} ,$$
$$F^{r}_{nc} = \frac{2}{\sqrt{1-\frac{2\lambda v^n}{r}\left(1-\beta^2\right)}} \left[\frac{\left(1-\frac{2\lambda v^n}{r}\right)\lambda v^n + \lambda n r v^{n-1}}{r^2} - \frac{\lambda v^n}{r^2}\right] \frac{dr}{dt} , \tag{13}$$

where F_{cent} is the centrifugal force and F_{nc} is the force which depends upon a velocity $\frac{dr}{dt}$ linearly. We know that when $n \geq 1$, then $\lim\limits_{r\to 0, v\to 0} \frac{dv}{dr} = X_0$, where X_0 is a positive real constant. Thus, to find the forces in the central naked singularity one should consider three cases:

1. $n \in [1, 2)$. In this case it is easy to see that we have infinity inertial forces:

$$\lim_{r\to 0,\, v\to 0} F^{r}_{cent} ,= \infty$$
$$\lim_{r\to 0,\, v\to 0} F^{r}_{nc} .= \infty \tag{14}$$

2. $n = 2$. In this case we have:

$$\lim_{r\to 0,\, v\to 0} F^{r}_{cent} = -\frac{1}{1-\beta^2}\left(\left(\lambda X_0^2 + 2\lambda X_0\right) ,\right.$$
$$\lim_{v\to 0,\, r\to 0} F^{r}_{ns} = \frac{2}{1-\beta^2} 2\lambda X_0 \frac{dr}{dt} . \tag{15}$$

We can see that in this case the radial inertial forces have a real positive finite value.

3. $N > 2$. In this case all inertial forces equall zero.

The strength of the central singularity

According to Tipler's definition the singularity is strong if the following condition is held (Nolan 1999; Tipler 1977):

$$\lim_{\lambda\to 0} \lambda^2 R_{ik} K^i K^k = \xi > 0 , \tag{16}$$

where λ is affine parameter, K^i is the tangent vector to the singularity, R_{ik} is the Ricci tensor. Besides, ξ must be finite. In the case of usual Vaidya spacetime the condition 1 gives for the expression ξ:

$$\xi = \frac{2\dot{M}(v)}{r^2} v^2 . \tag{17}$$

We are interested in the case $M(v) = \lambda v^n$. One can obtain:

$$\xi = \frac{2n\lambda v^{n+1}}{r^2} . \tag{18}$$

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Now we should consider the following limit (as in the previous cases we denote $\lim\limits_{v\to 0, r\to 0} \frac{dv}{dr} = X_0$):

$$\lim_{v\to 0} \xi = 2nv^{n-1}x_0^2 \ , \tag{19}$$

From this we can conclude that if $n = 1$, then we have gravitationally strong naked singularity. If $n > 1$, then $\xi = 0$ and we have only gravitationally weak naked singularity.

From this result we can conclude that in Vaidya spacetime, in the case of the naked singularity formation the inertial forces in this region are infinite when the singularity is gravitationally strong. When these forces have finite real value or are equal to zero, then the central singularity is gravitationally weak.

Conclusion

In this paper we have considered Vaidya spacetime when the mass function is $M(v) = \lambda v^n$. We found out that when $n \geq 1$, the result of the gravitational collapse might be the naked singularity. It means that there exist a family of non-spacelike future-directed geodesics which terminate in the central singularity in the past and the time of the apparent horizon is bigger than the time of the singularity formation. However, if there is such a family of geodesics, then particles can move along them and we investigated the question which inertial forces act on this particle when it escapes the naked singularity. We found out that these inertial forces are infinite when $n \in [1, 2)$ and finite when $n \geq 2$. However, the naked singularity is gravitationally strong only in the case when $n = 1$. When $n > 1$, it is gravitationally weak. Thus, we can conclude that the inertial forces which act on the particle when it escapes the singularity are finite only when the singularity is gravitationally weak and infinite when it is gravitationally strong. However, the weak naked singularity is not that interesting because the manifold can be extended through it. Thus, if we consider the black hole or the naked singularity formation in the case of Vaidya spacetime, one should consider the linear mass function $M(v) = \lambda v^n$, where λ is real positive constant. One should also note that only the linear mass function is the solution of the Einstein equation which violate CCC because it states that only the strong singularity must be always covered with horizon.

Conflict of Interest

The author declares that there is no conflict of interest, either existing or potential.

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